Berkeley Economic Review

TABLE OF CONTENTS

PROFESSOR INTERVIEWS

RESEARCH PAPERS

PEER REVIEW BOARD

SUBMISSIONS POLICY

Format: Please format your submission as a Microsoft word document or Latex file. We would prefer submissions single-spaced in size 12, Times New Roman font.
Length: Maximum five pages single-spaced for essays or op-eds, not including works cited page(s). No maximum for theses or original research papers.
Plagiarism: We maintain a strict zero tolerance policy on plagiarism. According to the University of North Carolina, plagiarism is "the deliberate or reckless representation of another's words, thoughts, or ideas as one's own without attribution in connection with submission of academic work, whether graded or otherwise." For more information about plagiarism – what it is, its consequences, and how to avoid it – please see the UNC's website.
Citations: Use MLA format for in-text citations and your works cited page. Include a separate works cited page at the end of your work.
Send to: berkeleyeconreview@gmail.com

DISCLAIMER

The views published in this journal are those of the individual writers or speakers and do not necessarily reflect the position or policy of Berkeley Economic Review or the University of California at Berkeley.

COPYRIGHT POLICY

STAFF


Anastasia Pyrinis
Editor-in-Chief


Katherine Sheng
Editor-in-Chief
*Layout & Design**


Antara Jha
*Finance**


Charles McMurry
*Membership**


Juliana Zhao
*Advertising**


Margaret Chen
*Peer Review**


Selena Zhang
*Outreach**


Simon Zhu
*Research & Editorial**


Vatsal Bajaj
*Professional Development**


Vinay Maruri
*Professional Development**


Yechan Shin
*Research & Editorial**


Gurmehar Somal
*Webmaster**


Alex Cheng
Assistant Editor


Ankita Kini
Advertising Associate


Anne Fogarty
Layout Editor


Bill Fang
Professional Development Associate


Camila Reimpell
Outreach Associate


Camille Hillion
Assistant Editor

STAFF


Cherng Chyn Loh
Peer Reviewer


Danielle Chen
Peer Reviewer


David Lyu
Assistant Editor


Emaan Siddique
Peer Reviewer


Evelina Bergström
Advertising Associate


Ihina Ganguly
Advertising Associate


Katherine Blesie
Staff Writer - Qualitative


Kelly Trinh
Professional Development Associate


Mae Zheng
Staff Writer - Qualitative


Marc Castillo
Peer Reviewer


Matheus Proença
Outreach Associate


Mengyao Wu
Peer Reviewer


Min Joo (Julie) Song
Peer Reviewer


Odysseus Pyrinis
Assistant Editor
Staff Writer- Qualitative


Olivia Gingold
Staff Writer - Qualitative


Parmita Das
Staff Writer - Qualitative


Patrick Tagari
Peer Reviewer


Reilly Olson
Advertising Associate
Professional Development Associate


Renata Avila
Peer Reviewer


Abhishek Roy
Staff Writer - Qualitative

STAFF

Rhea Mehta
Layout Editor

Rishi Modi
Outreach Associate

Sofia Guo
Peer Reviewer

Xu Wang
Peer Reviewer

* *Officers*

Additional Staff Not Pictured:

- Aimee Guo, Contributor
- Alessandro Ortenblad, Contributor
- Ammar Inayatali, Peer Reviewer
- Angel Y. Hsu, Contributor
- Avik Sethia, Professional Development Associate
- Dana Wu, Contributor
- Daniel Hoover, Contributor
- Danny Chandrahasan, Contributor
- Gabriele Buontempo, Peer Reviewer
- Karthik Vegesna, Contributor
- Krishna Mantirraju, Contributor
- Matthew McBride, Assistant Editor
- Noah Kaplan, Assistant Editor
- Owen Singh, Peer Reviewer
- Renuka Garg, Assistant Editor & Peer Reviewer
- Sahil Dang, Outreach Associate
- Sashyak Pattnaik, Contributor
- Shreyas Hariharan, Advertising Associate
- Vassilisa Rubtsova, Staff Writer - Qualitative & Advertising Associate
- Xiaoyu Liu, Contributor

Professor David Roland-Holst

Interviewed by **Vassilisa Rubtsova**

Interviewer: Hello. Thank you for meeting with me for this interview.

Prof. Roland-Holst: Of course.

Interviewer: Before we dive into the interview, could you give me an overview of the work and research you've done.

Prof. Roland-Holst: I'm an economics professor here at UC Berkeley and I got my PhD here. I've been teaching here since 2000. My main areas of work are economic development and environmental economics. I have worked in over 40 countries and I've got over 100 publications. My main areas of study now are climate policy in China and California, separately. They are both interesting theaters for this work. In the larger area of development and climate change, I work primarily in China, Southeast Asia, and Central Asia. Those are the regions I have studied for most of my career but I also did work in Africa and Latin America. My work with California came along with California's climate initiatives.

Interviewer: Clearly you have done a lot of work with sustainable development. Currently developed countries reached their developed status through the indiscriminate use of fossil fuels, are sustainable rates of development in currently developing countries competitive with rates of development through traditional means of development through the use of fossil fuels?

Prof. Roland-Holst: The simple fact is that fossil fuel use is relatively inexpensive if you do not account for the social cost of carbon. That makes it irresistible as a development tool. Our prosperity today in the West today is completely based on the Industrial Revolution. We domesticated carbon fuels and this allowed us to develop technologies that were beyond the imagining of our ancestors. There is no question that we are on a carbon and fossil fuel intensive growth trajectory. But fortunately, that does not need to be repeated in developing countries. Some of those countries have used fossil fuels to develop rapidly. We see that with China. China's growth is very carbon intensive since they have huge carbon reserves. Fossil fuels remain relatively inexpensive but the cost of more sustainable energy sources, in particular solar and wind renewables, have been plummeting in the last decade. Solar has fallen more than 70% and wind has fallen 50%. So they are coming into the range of the same cost as fossil fuels. Developing countries will be able to leap frog from fossil fuels to renewable energy sources. The tricky thing is overcoming the political economy of fossil fuel energy resources. Those resources are in the hands of very powerful stakeholders, not only in the West but also in developing countries. When political institutions of developing and developed nations have to choose between renewable energy and fossil fuels, it is not just on a cost basis. It is also adjusted for the political economy. In that case it becomes more challenging to overcome the use of fossil fuels for development. There are too many sunk costs in fossil fuels and wealth in those assets. If we look at coal, coal in the United States is about 60,000 FTE workers that need a different future for themselves. They have had chronic poverty and health problems for generations because of coal. The stubbornness of the coal issue is really about stranded assets in the energy sector. This is a much more difficult problem to solve than transition systems for fossil fuel workers such as coal workers. If we look at the assistance offered to farmers in America in comparison, offering assistance to coal workers would be cheap. Solving the issue of workers in these industries is relatively simple, the challenge is solving the utility and the stakeholder problem. In China by contrast its about the workers. China has 5 million FTE coal miners so that becomes a social policy consideration when you want to decarbonize the chinese economy. The technologies are there. China already has the largest renewable sector in the world already. I am optimistic that there will be this transition to renewables on a large scale for economic development since China has already recognized the public health crisis that comes from coal consumption which hinders economic development. Other developing economies can learn from this example.

Interviewer: We have all of these widely accepted profit maximization models that do not account for social cost of externalities and so everyone ends up paying for these externalities because companies are not absorbing the costs. This is a broad question but do you think it is possible to change or adjust our profit maximization models and current practices to include consideration for externalities? Is it possible to reform the study of economics?

Prof. Roland-Holst: Economics is trying to reform itself. Economists are very aware of the shortcomings. The younger generation of economists are focusing their energies on this. Everyone understands the imperfections of the models. The question becomes is economic theory dead? It is not, it is evolving to the complexities of reality. The parallel to this is astronomy. We began with Newtonian mechanics. Isaac Newton came up with the model of how the heavens work and made accurate, reliable predictions. But it was based on many assumptions such as a frictionless universe and no relativistic interactions but we now know that the universe is much more complicated. The theories have evolved. We need more Einsteins in economics who can help us understand how these imperfections can be not only recognized and understood but also rectified. If you want to represent the social interests in economics what kind of interventions can you devise to correct for market failures? But in general economists understand that the perfectly competitive paradigm is misleading in every important ways. It not only misleads us in terms of the perceptions of the outcomes but it also misleads individuals to do things that may not be good for society. And not just with pollution but also with inequality. Certain amounts of inequality is interesting because it perhaps motivates people but too much inequality is bad for everybody. It creates social risk and that imposes costs on society. These are areas where we need the public interest to be represented to correct for the imperfections in markets.

Interviewer: In China a lot of sustainable and climate change policy has been implemented out of sheer necessity, in the United states the full impacts of climate change and environmental degradation are yet to be fully felt. China obviously has a very different economy from the united states so what can the United States do to implement some of the changes that China has given the United States economy?

Prof. Roland-Holst: It begins with accepting the reality of climate change. Going back to the example of coal. Coal consumption should have ended yesterday. We need to find a way to make that possible even if that means compensating stakeholders. The problem is that there is no public and political will which is the key difference between the United States and China to take action.

We are a wealthy country so we can afford to both slow the process of climate change and adapt to the effects of climate change. My biggest concern is lack of political action. We don't necessarily need to model our changes after China we just need to begin to account for climate change.

Interviewer: Looking at California, we see this political will to take action. California has been largely successful in implementing climate change policy and sustainability. Do you think what has worked in California could work in other states?

Prof. Roland-Holst: In fact, if all states adopted California's existing policies pollution reductions would be going down seven times as fast. These are on the shelf policies but again there's a lack of political will.

Interviewer: So political will aside for a moment, do you think that California's policies work in other states?

Prof. Roland-Holst: It technically could work but the problem is that the stakeholders are different across different states so we would have to have some kind of compensation scheme. I do not expect those invested in conventional energy to give up without a fight but they do not need to fight if we come up with a valid compensations scheme. Studies show that the benefits significantly outweigh the costs so it's feasible to compensate. If we look at the amount we spend on things such as agricultural subsidies, this compensation would be cheap.

Interviewer: On the topic of California and climate change policy, Californian regulations include the gas tax. In many ways the tax is regressive and negatively impacts people who do drive long distances and who have older models of cars. How do we go about addressing the regressive nature of the tax?

Prof. Roland-Holst: The benefits are so great that we could compensate those who would otherwise be adversely affected and the way to do that is to subsidize more efficient energy technologies for them. We shouldn't subsidize fuel for low income families or those who drive long distances because it only encourages them to make the problem worse. Instead subsidize the technology choices they make and make more efficient technologies available. Obviously low income households in California cannot buy innovative technologies anytime soon. It takes time for these technologies to trickle down to the used car market where low income families buy their vehicles. However you can absolutely implement incentives that would make those cars more affordable. Right now, Volkswagen is preparing a platform for twenty four different models of low cost electric vehicles and they are targeting directly the entry level buyer. That is why the regressive tax issue won't be an issue much longer. Part of the problem is behavioral. We have plenty of technologies on the shelf to address climate change but in part people just don't like them. Elon Musk kind of realized this but unfortunately he started at the top of the market so most people cannot afford his sustainable technology. You can't get people to buy vehicles by lecturing them about fuel efficiency. It is not what motivates them; it is a much more visceral thing. We need to make these technologies things that people desire. As far as the adverse effects of regressive taxes we can correct those.

Interviewer: During the transition to sustainable practices, business' profits do take a hit. How do we converge sustainability with profits made without sustainable practices?

Prof. Roland-Holst: We must recognize the incentives and society has to take responsibility for those adjustments. I am a big advocate of green microcredit for enterprise. The problem is not so much business, its small businesses. Large corporations when they need new technologies, they either borrow money at the prime rate from large multinational banks or they issues bonds on Wall Street; there's a very low cost credit there. Small businesses adopt technologies by using their credit cards; they simply don't have access to the credit that large corporations do. So their cost of credit is much higher so they are much more concerned when they are told they need to adopt new technologies. The state needs to step up and offer subsidization for the appropriate technologies.

Interviewer: When it comes to larger corporations then, the argument against imposing environmental regulation is that they can "always go abroad." How do you argue against that?

Prof. Roland-Holst: They simply don't. There is no evidence to support that. It has been tested and there is no evidence that they go abroad [when facing environmental regulation]. There is no evidence on leakage, companies running away. Most companies need to be near their markets. Labor costs are the bigger issue not compliance costs. The evidence on energy compliance is now very old. Companies in Japan have been complying to strict environmental regulation for decades. Compliance costs are single percentages of total costs. Big cost differences can move a firm but these costs are not that big. Of course they will exaggerate the cost, because they are trying to fight the regulation. But when you look at the verdict that has been rendered by cap and trade mitigation is cheap.

Interviewer: So then if compliance costs are relatively low how do we get a company such as Amazon, that leaves a massive carbon footprint to change their practices without affecting their profits which are dependent on being able to ship and deliver products using enormous amounts of fossil fuels?

Prof. Roland-Holst: We have to change their business model in some way. They are good at what they do which is making profits so they will keep doing what they're doing. When in dialogue with people like [Jeff Bezos] it is important to keep in mind what their behavioral model is.They got where they are because they are good at making profits a particular way and if you ask them to do something else they're not going to be receptive of it. They might use other resources to create social good but they will be really wary of changing their success model. Behaviorally we have to understand how these people get stuck in patterns. Success gets them stuck in patterns. Government coming up with sticks and carrots can get them out of those patterns and steer them in the right direction. The government essentially translates social objectives into economic reward or punishment.

Interviewer: Ok well thank you for sitting down with me for this interview.

Prof. Roland-Holst: Of course.

Photo Credit: KINAXIS BLOG

Professor Christopher Walters

Interviewed by ***Parmita Das***

Interviewer: I'd like to begin by speaking to you about how your personal journey led you to Economics and then delve deeper into your research interests. Could you begin by telling me about your background and how it helped shape your academic focus and what experiences helped you find your passion for Economics?

Prof. Walters: Sure! I went into college thinking I was going to do more humanities-related disciplines. I always kind of knew I liked school, so I knew I was probably going to go to grad school or something, but I didn't know what. I was interested in History and Philosophy as an undergrad. I didn't take any Math my first couple of years, but then I sort of happened to take an Economics class by chance and I realized it was a way of answering a lot of the same social questions I was interested in studying in a more quantitative way, and that appealed to me as someone who had a little bit more Math that I felt like I wasn't able to use in my History classes, so I just started taking more and went from there.

Interviewer: What inspired you to research into school choice and charter schools?

Prof. Walters: That's a good question too. A part of that was opportunity. In modern Applied Microeconomics, it is very important to have very detailed data on people's choices and outcomes, so I was looking for an area where I could get a combination of the right data and the right question. In grad school I was sort of interested in labor markets broadly construed and how people accumulate the kinds of skills that they sell on the labor market, but there is a lot of different sub-questions under that. In my graduate classes, readings, and recent work in top journals in this area, I got interested in the combination of choices and experiments that were on the frontier of the education literature. So I would say the modern applied micro paradigm, especially the way that I was taught in graduate school, is that you need a good experiment to be able to say anything interesting about a social science question. By that I mean a setting where you have something that looks like a well-controlled or randomized comparison where some group of people get access to some program or opportunity and another set of people randomly don't. That's like an experimentalist view of research. But I'm also interested in, at least to some extent, theoretical models of how people make choices and how their choices are linked to the benefits of the programs that are available to them. And so looking at the charter school literature, it was mostly focused on evaluating, in a kind of causal sense, what are the impacts of charter schools and other school-choice programs like that on the people that participate using this institutional fact that, among those who apply to those programs, in a lot of cases the assignment happens by random lottery. And so we like that as social scientists, that's a well-controlled comparison and we're confident interpreting the difference between lottery winners and losers as the causal effect of getting into this school and attending this school. But I noticed reading those papers and working on a couple early versions of those myself, that there wasn't much analysis in that literature of which people were entering those experiments and why. So that's why I got interested in the topic. I was interested in modeling exactly who it is that's selecting into the opportunity to attend the opportunity to attend a different school than your default neighborhood option and how that decision is linked to the benefit of doing that, for the kid or for the family. So the combination of being attracted to the experimentalist, clean, causal identification you get from lotteries with the opportunity to model people's choices with the administrative data on who is and is not applying and what their backgrounds look like is what led me to my work on that topic.

Interviewer: What are some areas you are looking into now and how are you looking to collect your data?

Prof. Walters: A lot of my work is secondary analysis of existing data sets, either experiments that other people have run or administrative datasets that have something that looks like a quasi-experiment, like lotteries that I mentioned. I have a few different projects but most of them have that feature, in one way or another. I have a couple projects on the Head Start program, which is a public preschool program for poor kids in the United States. In that strand of my work, I'm reanalyzing a large-scale experiment that the Department of Health and Human Services ran on the Head Start program, where people were randomly admitted or not admitted to Head Start, and then trying to understand what we can learn from that about who benefits from the program and how that relates to choices to participate. And so that's a secondary analysis on an existing experiment that someone else ran. In my work on school choice and school assignment mechanisms, I'm using administrative data on people's educational decisions and school enrollments that's generated as part of the natural process of managing a large, urban school district and figuring out who's going to what school and what their outcomes look like. The way I'm collecting most of my data is opportunistic in some sense - it's like data that's generated and out there in the world, either by previous experiments or by government bodies that are implementing or managing programs - and I'm looking for opportunities to use that sort of data to answer questions about effects of programs on people's outcomes.

Interviewer: We learned in Econ 2, a basic Economics class, that the return on investment in human capital lowers as a person progresses through their education. So, do you think the outcome or decision-making mechanism would change for that person, and would differ from the work you did on charter schools for example?

Prof. Walters: I'm not sure I totally agree on the premise of that question. That question is premised on the idea that the return on human capital investment is largest early-on in the schooling year. There's certainly a lot of evidence that highly effective preschool programs have very large social returns. And I think that evidence is convincing, but I think there's also more recent evidence that even at later stages in their career - like middle and high school, or even college - there's pretty large returns on human capital investment as well. For example, for marginal college students in the United States, in my view, some of the best evidence suggests that the return to year of college for students at the margin between attending a four-year college and not is something on the order of 10% per year or higher. I'm thinking of some research by Seth Zimmerman, who's an economist at the University of Chicago School of Business. I'm not sure all economists would agree with me, but I think our best evidence suggests there's actually pretty large returns to human capital investment at all different stages of the educational career, including the college attendance decision.

Interviewer: So what made the choice of subfield in Economics clear for you? What made you decide on Labor Economics as your focus?

Prof. Walters: I think my choice to focus on Labor instead of other subfields of Economics is a combination of the set of questions you get to answer in Labor and the sort of research philosophy of the field, which are linked to each other. The questions that Labor economists focus on are very intimately linked to actual, concrete measures of well-being in people's lives-their wages, their employment outcomes, what their careers look like. It's very practical and concrete, and not very abstract. I was kind of attracted to that set of questions; answering questions about real sources of well-being or lack thereof in people's lives. I think because of that focus on those sorts of questions, Labor is also, from a methodological perspective, a very practical field. We're interested in developing methods that can actually be used in real datasets to answer important policy questions, and I was attracted to those methods as well, in addition to the questions.

Interviewer: So what made the question of "Industry or Grad School" clear to you?

Prof. Walters: I'm not sure. I never had a real job and I felt like I was pretty good at school, and I decided I was gonna keep doing it.

both laugh

Interviewer: That's a fun answer. It was a pleasure to interview you. Thank you for your time!

[THIS PAGE INTENTIONALLY LEFT BLANK]

Professor Gabriel Zucman

*Interviewed by **Katherine Blesie***

Interviewer: Many of our readers are aspiring economists, and I'm sure they would be curious to hear about how you got started in the field of economics.

Prof. Zucman: I started my PhD during the financial crisis of 2008. I wanted to understand what had caused this dramatic economic event and also what types of government policies could contribute to more financial stability and sustainable, equitable growth. Because I wanted to understand what was going on I started looking into macro-economic data, data on capital flows in particular. It's when you're looking at this data that you see hundreds of billions of dollars flowing in and out of places like the Cayman Islands, Bermuda, tax havens like that. I thought wow, that's interesting, that's something we read about in the newspapers, but that academic economists don't talk a lot about.

Interviewer: Why do you think it wasn't being talked about?

Prof. Zucman: That's a good question. It's a mixture of reasons. One is that there was very little data, about tax havens in particular, and economists have a tendency to study phenomena for which there is actually data, which is perfectly understandable, but it also means that there are important phenomena occurring like tax that we don't think about enough from an economic standpoint. Something like tax evasion is never going to have perfect data by its very nature, and so you have to come up with indirect methods and way to quantify it. Also, for a long time economics was very theoretical, mostly based on models and theories and with not a lot of serious effort to look at data. That changed in the 1990s, beginning of 21st century. The third factor is that for a long time economists did not care a lot, and certainly not enough, about inequality. They mostly cared about representative agent models based on the fact that from 1950 to 1980 growth had been relatively equally distributed, and so they felt that the thing of first order importance, the thing that was really important was growth, and not inequality.

Interviewer: So why is it important that we study inequality in economics?

Prof. Zucman: It's important because the headline macroeconomic statistics these days don't reflect at all the dynamic of income for the vast majority of the population. For instance, since 1980 there's been about 1.4% growth a year on average in the US, but for half of the population there's been zero growth. And so you just cannot understand what's going on in the economy if you just focus on the macroeconomic aggregates. So even if you just want to understand how the economy functions, you want to have disaggregated data, you want to have data on income, and not on average income but on income for the various groups of the distribution, you want to have data on growth, and not on average growth but on growth for the various groups of the distribution. And then people care about inequality because they feel that it's something that matters politically. Human beings compare themselves to others. They care about extreme inequality because it can affect the politically process, the policy making process. If wealth is too concentrated in just a few hands it means that the politically process is more likely to be controlled by a few very wealthy individuals. So, if you care about democracy, the type of society we live in, then you care about inequality.

Interviewer: You've said in the past that extreme wealth concentration, or, in fact, wealth concentration to any affect, can affect democratic institutions. Can you expand on this a little?

Prof. Zucman: Wealth is power, and so a huge concentration of wealth means a huge concentration of power. A small, extremely wealthy, fraction of the population has the power to greatly influence the policy making process. It's not that the very wealthy are evil people. It's that they don't need public schools, or public hospitals. They just don't need these things, and so some of the very rich just don't want to pay taxes to support them. If things are more equally spread, if we all need public schools and a public healthcare system, then it makes these institutions more sustainable.

Interviewer: Are you of the opinion that a universal basic income would be a band-aid fix for a larger systemic problem?

Prof. Zucman: It's a complicated question. One of the specificities of the US as a country and the US system of government, taxes, and transfers, is that there is no support for people who don't have kids, people who are unemployed. Because there is no safety net, you have a fraction of the population that earns no income, and lives in dire poverty. Most developed countries have a minimum income, a safety net, where nobody can earn less than x. From that perspective, I think introducing a universal basic income in the US would be a very good thing. It's a missing component in the US welfare state, the US safety net. There are many missing components, but that's one, an important one. Now, we can debate the merits and demerits of how a universal basic income should be structured, whether you want to structure it in a way where you send checks to everybody, etc. Should we exclusively rely on a universal basic income when dealing with rising inequality? That, I think, is a mistake. The countries that have successfully addressed inequality, reduced inequality, did so primarily not through government transfers, but by equalizing the distribution of market income, of pre-tax income. In particular, in making sure that everybody can work and earn a decent wage. That, historically, and throughout the world, has been the way that inequality has been contained. Because if you have extreme market inequality, and then, okay, the wealthy pay taxes, that fund transfers for the poor, that's not very sustainable. At some point, you know, the rich start to feel something along the lines of I'm so deserving of my high income, why am I paying for people that don't work? That's not sustainable. So the short answer is: a universal basic income is probably a good idea, given that it's a missing component of the safety net, and the welfare state, but it's far from enough.

Interviewer: What should the primary focus of tax policy be? Should it be on bolstering the middle class, or propping up the bottom 10%?

Prof. Zucman: I think these things go together. If you tax the rich it affects the entire distribution, because it changes the incentives for very rich people to earn a lot of income, to extract rent at the expense of other parts of the distribution. If you have top marginal income tax rates of 90% or more, like the US had for a very long time, then it completely changes the incentives of corporate executives to pay themselves very high salaries, so that means more money that can be used to pay ordinary workers more, for instance. I don't think you can think about these various groups in isolation. I think tax policy affects everything, and that's what the historical record shows, that when the top tax rates were very high income growth rates for the middle class and for the working class were higher. That's why I think it makes sense to say: okay, let's change taxation - of course you're not going to fix all the problems - but progressive taxation is a very powerful economic policy because it affects all taxable behavior. You can regulate finance, and of course that's very important, you can regulate health care, which takes of 20% of GDP and is of course very important. But progressive taxation changes behavior across the board, in all sectors of the economy, and so it's the most powerful and the most direct way to change the dynamic of income growth across the spectrum.

Interviewer: Do you think the most valid critique of progressive taxation is that it will increase the incidence of tax avoidance?

Prof. Zucman: Too many people have the view that tax avoidance is an unavoidable given, and that if taxes are higher people will avoid or evade more. That's not true. That's not what the data shows. Tax avoidance and tax evasion are things that government police can affect. If you have a tax system that has few or no loopholes, then you cannot avoid taxes. If you create lots of loopholes, then people are going to use them. That's a choice that governments make. You can choose to have no loopholes. You can choose to spend resources to enforce the law, and fight tax evasion, in terms of collecting information, implementing sanctions against financial institutions or the countries that facilitate tax evasion. All of this can be controlled through appropriate tax policy. My view is that if the US moved towards a more progressive tax system then also, logically speaking, we would see more of that effort to curb tax evasion and a more concerted effort in reducing loopholes, making sure it's not possible to legally avoid taxes. If it's done that way then tax reform can work.

Interviewer: How would this kind of effort deal with multinational corporations like Apple exploiting foreign countries for tax evasion?

Prof. Zucman: That's a good illustration of how the US government has let big US companies shift profits to offshore tax havens, and accumulate trillions in untaxed earnings in places like the Cayman Islands, Bermuda, Ireland. That was a choice that was made, that could have been done differently. One way to do it differently would be to say if Apple, for instance, makes 50% of its worldwide sales in the US, then the US is going to consider that 50% of their worldwide profits have been made in the US. That's going to be the tax base, that's what's going to be taxed in the US. The beauty of this is that Apple today can manipulate the location of their profits but they can't manipulate the location of their customers. Their customers are in the US. Such a system makes tax avoidance impossible. It illustrates this very simple idea that tax avoidance is a policy choice. With a different tax system we can reduce this tax avoidance to zero. It's doable.

Interviewer: Can Senator Warren and Representative Cortez' tax proposals work in conjunction to reduce inequality?

Prof. Zucman: Yeah, I think they can work together, and I think most people agree that the proper way to tax the rich is not with just an income tax, or just a wealth tax, or just an estate tax. You need a wealth tax because many very rich people, billionaires, have a ton of wealth but very little taxable income. For them, even if you increase the top marginal income tax rate to 70% it's not going to make any difference to how much tax they pay in total. Whereas with a wealth tax, well that's going to change a lot. You want an income tax because you have people who earn a lot of income, like corporate executives, but maybe don't have a lot of wealth, and so the proper way to tax them is through the income tax. You want a progressive estate tax because if you care about meritocracy, if as a society we prefer self-made wealth over inherited wealth, then you want to tax inherited wealth more through the estate tax. These three things complement each other. I think that ultimately, the proposals that will be made during the primaries will reflect this.

KB: What has been the response to your work in the sphere of public policy?

Prof. Zucman: I think, by now, the vast majority of the population, including the vast majority of economists, realize that rising inequality is real, and is a very serious problem and that something needs to be done. I think the idea of wealth taxation has become much more mainstream than it was even a few years ago. I think 5 years ago majority of economists would have said it doesn't make sense, we already have an income tax, et cetera. Now that we have better data on wealth concentration and a clearer understanding of how the wealth tax would work, how it would be implemented, I think - I don't know if it's the majority of economists - but I think a big fraction of economists and also a big fraction of people in the policy making world, in Washington D.C., find the idea of a wealth tax perfectly sensible. I think it has, actually, a pretty bright future.

Interviewer: So you're optimistic for the future?

Prof. Zucman: I'm pretty optimistic, yes.

Photo Credit: PACIFIC STANDARD

Curbing or Displacing Deforestation? The Amazon Blacklist Policy

João Pedro Vieira
University of California, Berkeley

Abstract

This paper tries to establish causality between the Amazon blacklist policy and deforestation displacement, by applying the differences-in-differences framework, on a panel of Cerrado Municipalities from 2004 through 2014. The results are statistically significant, showing evidence of displacement at intermediate distances (50-200 km). These findings are robust to different treatment cut-off definitions. Also, the parallel assumption holds when looking at the pre-trends. Counterfactual simulations show an increase in deforestation of 4,963 km2 from 2009 through 2014, representing an offset of 29% of the direct impact of the policy.

Keywords

Displacement, Deforestation, Differences-in-Differences, Amazon Blacklist, Cerrado

Acknowledgements

I thank Professor Assunção, Professor Eichengreen, Matthew Tauzer, and Stephanie Bonds for the continuous support and feedbacks as my advisors. I am also grateful for the contributions of Professor Pouzo, Professor Werneck, Deepak Premkumar, Clarissa Gandour, Flávio Romero, Guilherme Jardim, Helena Arruda, Renata Avila, and Tomás Do Valle. All of the remaining errors are my responsibility.

Table of Contents

1 Introduction

The Amazon is the largest biome of Brazil, covering an area of 4,106,943 km2, followed by the Cerrado that covers 2,036,448 km^2 (IBGE, 2004). However, when recent and historical deforestation rates are compared, the positions are reversed. Even though the Cerrado is considered one of the world's biodiversity hotspots (Myers et al., 2000; Silva and Bates, 2002), by the early 2000s, almost half of its total area had already been converted to pasture or cropland (Klink and Machado, 2005; MMA, 2015; Noojipady et al., 2017). Whereas, in the Amazon, this share reached only 18.2% in 2013 (Nobre, 2014). Moreover, for the 2004 through 2014 period (excluding 2005), deforestation rates were consistently higher in the smaller biome.

This picture supports the estimates of Noojipady et al. (2017), that two-thirds of the Brazilian greenhouse gas emissions were due to changes in land use and forest loss in 2005. To stop this trend, the Brazilian government launched an integrated action plan (PPCDAm[1]). The plan focused on the preservation of tropical forests typically present in the Amazon, even though the deforestation pressure was higher in the Cerrado.

The conservation policies' turning points coincide with the sharp falls in deforestation rates for the Amazon. However, as pointed out by Noojipady et al. (2017), to achieve national emission reductions, it is necessary to take into account possible cross-biome leakages. For example, between 2010 and 2013, they estimated that carbon emissions due to land use changes in Cerrado offset 5% to 7% of the avoided emission from the Amazon.

This paper explores the institutional changes in the mid-2000s that focused conservation efforts in the Amazon over the Cerrado. Specifically, the focus is the Priority Municipalities' (PMs) policy which created a blacklist, in 2008, of municipalities with high clearing rates. The question to be answered is "Did this policy displace deforestation from the Amazon to the Cerrado?"

1 Action Plan for the Prevention and Control of Deforestation in the Legal Amazon

The rationale is that when a municipality enters the blacklist, there is an exogenous rise on the cost of deforestation due to strict law enforcement (Assunção and Rocha, 2014) or other non-enforcement mechanisms (Cisneros et al., 2015), leading to an incentive to displace. Additionally, considering the focus of the policies, the Cerrado seems to be a more attractive region compared to non-blacklisted Amazon municipalities.

Based on a panel of Cerrado municipalities from 2004 through 2014, I use a differences-in-differences framework, considering municipalities that are less than 300 km from the closest PM as treatment. The model indicates a statistically significant increase in the farming area, a proxy for deforestation, at intermediate distances from 50 to 200 km. In robustness checks, I find supporting evidence for the parallel assumption, looking at the pre-trends. Also, I verify that the results are not driven by the arbitrary treatment cut-off by testing a 250 km threshold. Finally, counterfactual simulations suggest a total displacement of 4,963 km2 from 2009 through 2014. That represents an offset of 29% of the avoided deforestation in the targeted Amazon municipalities, compared to the direct impacts estimated by Assunção and Rocha (2014).

This study is closely related to two main pieces of literature: crime literature and literature that evaluates the impact of the mid-2000s Amazon anti-deforestation policies.

From crime literature, I am interested in the debate about hotspot policing and displacement or diffusion effects. Chalfin and McCrary (2017) define hotspots policing as a reallocation of existing resources to places where crime is highly concentrated. The question that follows is if this strategy merely shifts, through the displacement effect, rather than reduces, crime. Weisburd et al. (2006) point out that for a long period of time, it was believed that displacement was inevitable; however, now many critics don't think that is the case (Barr and Pease, 1990; Gabor, 1990; Eck, 1993; Hesseling 1994; Clarke, 1995). Moreover, Clarke and Weisburd (1994) show that the phenomenon of "diffusion of benefits"—the reduction of crimes in areas outside the targets of intervention and considered the reverse of displacement—is also possible. Since there is mixed evidence of the presence and direction of spatial spillovers, documenting these effects remains an empirical challenge.

Most studies about anti-deforestation policies focused on direct impacts of the policies (Hargrave and Kis-Katos, 2013; Arima et al., 2014; Assunção and Rocha, 2014; Assunção et al., 2015; Cisneros et al., 2015; Assunção et al., 2017; Burgess et al., 2018), leaving spillover effects as a by-product (Cisneros et al., 2015; Assunção, Gandour and Rocha, 2017) when analyzed. Additionally, even when the focus was displacement, the sample used was geographically restricted to the Amazon Biome (Amin et al., 2015; Andrade, 2016). In general, this literature documents that the anti-deforestation policies were the main drivers of the observed slowdown in deforestation in the Brazilian Amazon during the mid-2000s, with small negative spillovers (Amin et al., 2015) or even positive externalities (Andrade, 2016).

Hence, my main contribution is prioritizing the identification of a spillover effect of one of the anti-deforestation policies, focusing on the much less explored Cerrado region. Additionally, I provide new evidence for the debate about hotspots policing from crime literature, applying it to the less explored setup of environmental crimes rather than urban crimes.

The rest of this paper is organized as follows: Section 2 discusses the Amazon anti-deforestation policies and the differences between the Cerrado and the Amazon; Section 3 provides a description of the data; Section 4 explains the empirical strategy used to estimate the spatial spillover effect; Section 5 discusses the results of the paper; Section 6 provides robustness checks for the model assumptions; and Section 7 concludes by summarizing the results and presenting its policy implications.

2 Institutional Context

In the early 2000s, deforestation in the Brazilian Amazon rose until a peak of 2.8 million km² in 2004. As a response, the Brazilian government created an integrated plan of action (PPCDAm) with the goal of proposing new approaches to curb deforestation in the Legal Amazon.[2] The two main reformulations were the use of a satellite-based system to detect tropical clearings and the creation of a blacklist of the municipalities in need of special attention.

The first phase of PPCDAm started in 2004, and its main component was the strengthening of monitoring and law enforcement. Since 1989, Ibama[3] is responsible for addressing environmental violations acting as the national police authority, and, until 2004, their actions were mostly based on voluntarily anonymous accusations of illegal activities. After 2004, however, there was a massive advance in the identification process of clearings in the Amazon, due to the adoption of DETER[4], developed by INPE [5]. This system processes forest cover images in 15-day intervals, comparing the same area across time to identify signals of forest loss, and then issuing alerts with the location of the threatened areas. In practice, when the offenders are caught red-handed, they can be punished more efficiently, so timing is fundamental and DETER allowed Ibama to act more quickly (Gandour, 2018).

In 2008, the second phase of PPCDAm was initiated, marked by significant legal changes. First, the Presidential Decree 6,514 regulated the use of penalties like fines, embargoes, and seizure and destruction of equipment as punishment of environmental crimes (Brasil, 2008). Additionally, the Presidential Decree 6,321, signed in December 2007, allowed the exposure of municipalities with intense deforestation in recent years. The selection criteria to be included in the list of Priority Municipalities (PM) were: (i) total deforested area; (ii) deforested area over the past three years; and (iii) increase in deforestation rate in at least three of the last five years (Brasil, 2007). The first list was released in 2008 with thirty-six PMs, seven more were included in 2009 and 2011 and two more in 2012.

The primary mechanism of action was the adoption of a hotspot policing strategy that focused the attention of the Law Enforcement on areas with high crime rates. With a larger share of dedicated Ibama resources, alerts issued in these areas were prioritized, private land titles were revised, and licensing, georeferencing requirements, and authorizations for clearing in rural properties were made harsher (Assunção and Rocha, 2014). Furthermore, other non-command and control mechanisms became anecdotally documented, like the punishment in the polls of local politicians and the refusal of the supply chains to buy cattle from embargoed areas (Abman, 2015; Cisneros et al., 2015).

This policy has received some attention from the impact evaluation literature. Regarding direct impacts, there is evidence of a significant reduction in deforestation for these areas (Assunção and Rocha, 2014; Arima et al., 2014; Cisneros et al., 2015). For the mechanism of impact, Assunção and Rocha (2014) argue that law enforcement fully explains the reduction, while Cisneros et al. (2015) estimate that non-enforcement mechanisms account for the impact. For spatial spillover effects, Cisneros et al. (2015) find no evidence of either deterrence or displacement effects using a combination of matching and double difference frameworks. On the other hand, Andrade (2016) uses a spatial differences-in-differences model and estimates a significant and economically relevant deterrence effect, showing a reduction in forest clearing for non-blacklisted municipalities with PM as neighbors. Note that both studies look only to tropical deforestation, which excludes the majority of the Cerrado Municipalities. Therefore, this paper aims to fill this gap by focusing on spatial spillovers looking at land use changes in the Cerrado.

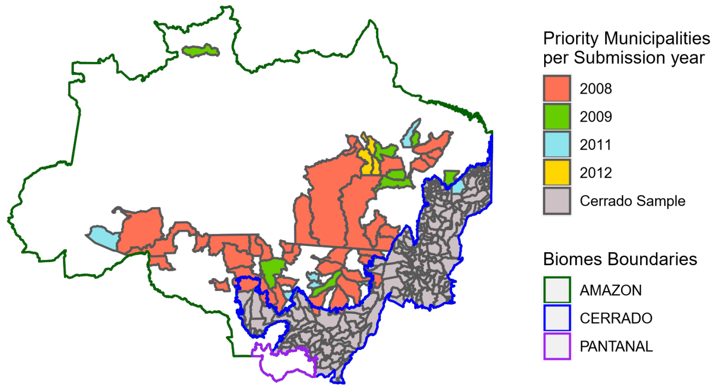
2 Legal Amazon is a geopolitical division of Brazil, includes the whole Brazilian Amazon Biome, part of the Cerrado and the Pantanal
3 Brazilian Institute for the Environment and Renewable Natural Resources
4 Real-Time Detection of Deforestation System
5 National Institute for Space Research

The Cerrado might be an interesting study case and reveal different findings because most of the policies implemented in the mid-2000s were aimed at the Amazon Biome and increased the difference between the cost of deforestation in these two areas. First, the forest code requires 80% of conservation in private properties in the Amazon Biome, while for the Cerrado it requires only 35% when inside the Legal Amazon and 20% outside it (Brasil, 2012). Secondly, the innovative monitoring system (DETER) only detects tropical clearings, thus excluding the majority area of the Cerrado that is composed by savanna-vegetation. Lastly, almost half of the Amazon Biome is considered a Protected Area, while this share is only 18.6% in the Cerrado (Nobre, 2014). As a result, Cerrado municipalities seem to be much more attractive for displacement compared to non-blacklisted Amazon municipalities.

3 Data

The empirical analysis is based on a municipality-by-year panel dataset built from multiple publicly available sources, from 2004 through 2014. The sample includes the Cerrado biome of all the municipalities inside the Legal Amazon comprising 355 municipalities. Figure 1 shows the Legal Amazon region with the sample in grey, the biomes spatial boundaries, and the blacklisted amazon municipalities colored by the year of submission. This section briefly describes the variables used in the analysis. More details about the construction process and about the data sources are given in Section 9.1 (Appendix).

Figure 1: Map of Cerrado Sample and Amazon Priority Municipalities



Note: The figure maps the Brazilian Legal Amazon and biomes spatial boundaries. The gray region indicates the spatial sample for the analysis, defined as the area inside the Cerrado biome. The colored municipalities are the ones in the blacklist, color varies accordingly with the submission year.
Data sources: IBGE, MMA, MapBiomas.

3.1 Farming

Deforestation in the Cerrado is not as simple to classify as deforestation in the Amazon, so to overcome the data limitation I calculated the share of municipal area destined for farming to be a proxy for deforestation. As shown by Noojipady et al. (2017), 88% of the forest loss was destined to farming in the Cerrado. Therefore, these variables are closely related.

3.2 Treatment: Distance Criteria

The treatment definition is based on the distance to the closest PM of the 2008 list. If the distance is less than 250 km, the unit is part of the treatment group, and if it is above 250 km, it is part of the control group. In a robustness check, I change this cut-off from 250 to 300 km. To allow spatial heterogeneity, the treatment group is divided into five subsets based on distance intervals of 50 km.

3.3 Agricultural Commodity Prices

Following Assunção et al. (2012), I use an exogenous commodity price series with annual international prices for corn, soybean, rice, sugarcane, and cattle. Then, each crop is weighed based on the share of the municipal area used as farmland for production averaged from 2000 through 2003. Moreover, for cattle, the ratio of heads of cattle to the municipal area is used for the same period. I use the period pre-sample to avoid endogeneity issues due to changes in production as a result of the policies starting to be implemented in 2004.

3.4 Weather Control

Based on the literature that forest loss can affect a region's microclimate (Nobre et al., 1991; Aragão et al., 2008), and that meteorological conditions can also affect land use decisions, controls for annual average temperature and annual total precipitation are added.

3.5 Policy Control

Finally, I also use variables that capture the presence of other policies as the share of the protected area and a dummy for being a Priority Cerrado[6] municipality. These policies might be admittedly endogenous because they probably are affected by the treatment. Thus I only use them for robustness purposes.

3.6 Summary Statistics

Tables 3 and 4 in Section 9.2 present the means and standard deviations by year of the variables used in the empirical analysis.

6 A Priority Municipality list was created for the Cerrado in 2012

4 Empirical Strategy

4.1 *Model*

The proposed empirical strategy aims at exploring how the implementation of the Amazon Blacklist changed the land use trends in near Cerrado municipalities, which indicates the presence of displacement effects. I draw on a differences-in-differences framework to infer causality. The benchmark equation is:

$$Farming_{i,t} = \sum_{break=0-50km}^{200-250km} (\rho_{break} * Treat\_break_i * After_t) + X'_{i,t} * \omega + \alpha_i + \theta_t + \varepsilon_{i,t} \quad (1)$$

where $Farming_{i,t}$ is the fraction of the municipality i destined for farming in year t; break comprises five distance intervals (0-50 km; 50-100km; 100-150 km; 150-200 km; and 200-250 km); $Treat\_break_i$ is an indicator that equals 1 when the distance from muni i to the closest PM is contained in the *break* interval; $After_t$ is an indicator equals 1 when year t is greater than 2008; $X_{i,t}$ is a vector of muni-level controls for weather, agricultural prices and observed policy; α_i and θ_t are, respectively, municipality and year fixed effects; εi,t is the muni-year idiosyncratic error. Estimates are robust to heteroskedasticity, and standard errors are clustered at the municipality level in all specifications, making them robust to intra-municipal serial correlation (Bertrand et al., 2004). ρ_{break} are the differences-in-differences estimators of the spillover effect for each distance *break*. It is also relevant to notice that the control group, in this case, is the omitted *break* category composed by all the municipalities more than 250 km distant from the PMs.

4.2 *Identifying Assumption*

The fundamental identifying assumption in the differences-in-differences framework is that the control group trend is a valid counterfactual for the treatment group trend in the absence of treatment. One can never directly test it since only one potential outcome is observed each year. However, to get confidence that this assumption holds, I inspect the trends of the treatment and control groups when they both have the same treatment condition, for example before the policy, in Section 6.1.

Looking at the pre-trends can give us confidence, but certainly does not pin down the identification. One might still argue that the control group is also being affected by the policy, or that, after the policy, variables relevant to land use decisions might have changed in ways that made the treatment and control group trends diverge for reasons not associated with the policy itself. For the former, I argue that 250 km, a considerable amount of distance, still is an arbitrary cut-off, so in Section 6.2 I check if our results hold using a more conservative cut-off of 300 km. For the latter, I not only add year and municipality fixed effects, controlling for all time-invariant and unit-invariant variables but also control for some covariates that vary across time and municipalities to mitigate omitted variable bias.

5 Results

5.1 Main Results

Table 1 provides the estimated coefficients of the displacement effects for each distance break. All specifications include municipality and year fixed effects.

Table 1: Distance Breaks Regression Results

VARIABLES	(1) Farming	(2) Farming	(3) Farming	(4) Farming
After x Treat (0-50km)	0.01034***	0.00980***	0.00904**	0.00905**
	(0.00364)	(0.00363)	(0.00354)	(0.00377)
After x Treat (50-100km)	0.01622***	0.01540***	0.01448***	0.01468***
	(0.00466)	(0.00457)	(0.00449)	(0.00461)
After x Treat (100-150km)	0.01224***	0.01190***	0.01073***	0.01078***
	(0.00330)	(0.00332)	(0.00329)	(0.00333)
After x Treat (150-200km)	0.01834***	0.01825***	0.01671***	0.01671***
	(0.00391)	(0.00388)	(0.00370)	(0.00368)
After x Treat (200-250km)	0.00632	0.00628	0.00492	0.00490
	(0.00518)	(0.00518)	(0.00510)	(0.00510)
Observations	3,905	3,905	3,905	3,905
Number of Municipalities	355	355	355	355
R-squared	0.21011	0.21244	0.22627	0.22655
ctrl FE	yes	yes	yes	yes
ctrl weather	no	yes	yes	yes
ctrl prices	no	no	yes	yes
ctrl policy	no	no	no	yes

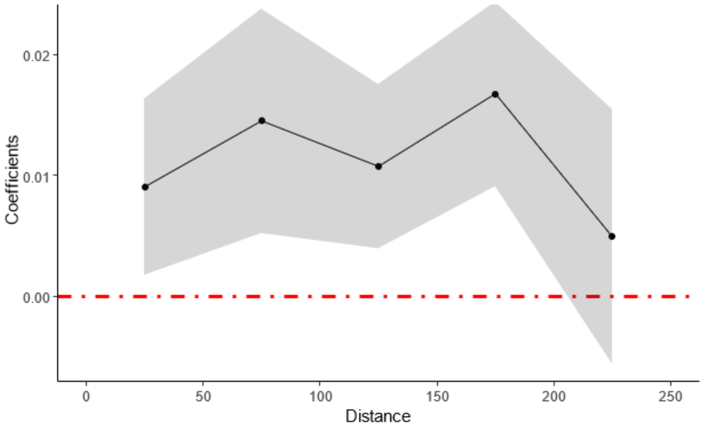
*Notes: The table reports fixed effects coefficients for Equation 1 (Section 5.1). The dependent variable is the share of the municipal area destined for Farming. Reported independent variables are the diff-in-diff estimators. After is a policy indicator = 1{year > 2008}. Treat (break km) are treatment indicators = 1{DistancetotheclosestPM ⍰ break}. The control group is the omitted category 1{Distance > 250 km}. Controls are added gradually to the specification. The no/yes markers in bottom rows indicate the inclusion of the following sets of muni-level controls: (i) muni and year fixed effects; (ii) climate: precipitation and temperature; (iii) weighted agricultural prices: cattle, corn, soybean, rice, and sugarcane; and (iv) observed policy: share protected area and cerrado priority municipality status. The muni-by-year panel includes 355 municipalities located in the Cerrado biome within the Legal Amazon and covers 2004 through 2014 period. Standard errors are robust and clustered at the municipality level. Significance levels: *** p<0.01, ** p<0.05, * p<0.10.*

Column 1 controls only for fixed effects. Column 2 adds temperature and precipitation controls. Column 3 adds weighted agricultural prices including, cattle, soybean, rice, sugarcane, and corn. Column 4 adds observed policies including the share of protected area and a dummy for Cerrado priority municipality.

The preferred specification is Column 3 because it uses all the sets of controls, except for the one that might be endogenous. These results corroborate the hypothesis of displacement effects. For municipalities between 50 and 200 km, the policy had a positive and significant impact at the 1% level, generating an increase in the range of 1.07 and 1.67 percentage points in the fraction destined for farming, representing a deforested area of similar magnitude. For municipalities less than 50 km away there is still a significant impact, but at the 5% level and smaller in magnitude. That can be explained by the fact that when offenders are very close to areas with hotspot policing they might perceive an increase in the cost of illegal deforestation, thus reducing their activities and attenuating the displacement effect. Also, it can be inferred that the displacement reach is 200 km, considering that the coefficient for the 200-250 km break is not statistically different from zero at any usual significance level. By looking across the columns it is clear that the coefficients are stable, serving as evidence that they represent a causal impact.

Figure 2 represents the coefficients from Column 3 of Table 1 graphically, showing how the impact of the policy varies spatially with a 95% confidence interval.

Figure 2: Distance Breaks Regression Coefficients



Notes: The graph plots the fixed effects coefficients from our preferred specification (Table 1, column 3) and the shaded area is the 95% confidence interval.

5.2 *Counterfactual Simulation and Economic Impact*

To assess the economic impact, I propose a counterfactual exercise setting all the treatment variables to zero. Consequently, I simulate a scenario with no implementation of the blacklist policy, so I calculate the predicted value of the farming area by multiplying the farming share by the municipality area (Estimated Farming Area), and finally doing the same for the baseline model with the observed data (Observed Farming Area).

Table 2: Counterfactual Exercise - No Blacklist Policy

Year	Observed Farming Area	Estimated Farming Area	Difference observed-estimated
2009	282,180	281,225	955
2010	284,571	283,666	905
2011	287,324	286,369	955
2012	291,189	290,081	1,108
2013	293,710	292,643	1,067
2014	295,251	294,278	973
Total 2009-2014	1,733,225	1,728,262	4,963

Notes: All areas are in square kilometers. The counterfactual simulation is conducted using estimated coefficients from our preferred specification (Table 1, column 3). The hypothetical scenario sets the treatment interaction terms from 2009 through 2014 as zero to capture the complete absence of the blacklist policy. Observed Farming Area shows total recorded sample area destined for Farming; Estimated Farming Area shows total estimated sample Farming Area in the hypothetical scenario; Difference reports the difference between observed and estimated totals.

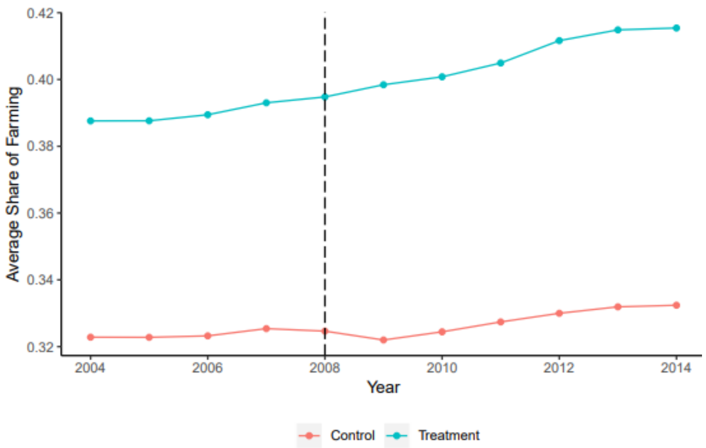
Comparing these two annual results (Difference observed-estimated), I calculate an increase of 4,963 km^2 due to displacement effects, for 2009 through 2014, in the sample. Then I compare it to the direct impact, estimated by Assunção and Rocha (2014), of 11,396 km2 of avoided clearings due to the same policy. To make the comparison more similar, I use the average displacement per year: 827.17 km^2 and the direct impact per year: 2,849 km^2. Using these numbers, I calculate that the cross-biome leakage generated an offset of 29% in the policy impact.

6 Robustness Checks

6.1 Parallel Assumption Tests

To get a visual notion of the trends, I calculate the average farming share for treatment and control groups for each year and plotted it as shown in Figure 3.

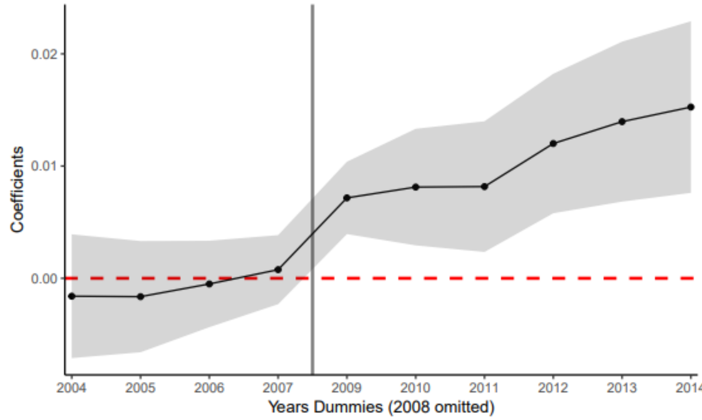
Figure 3: Visual Inspection of Parallel Trends



Notes: The graph plots the trends of the average area destined for Farming for treatment and control groups for the period 2004– 2014.
Data Sources: MapBiomas

As shown above, the pre-trends are very similar. However, to formally test the parallel assumption for the pre-trends, I use the leads and lags regression. This model requires a treatment time dummy for each year before and after policy implementation (2008), and the same set of covariates is used as a control as in the preferred specification from column 3 of Table 1. Figure 4 represents the coefficients for the time dummies graphically.

Figure 4: Parallel Assumption Test - Leads and Lags



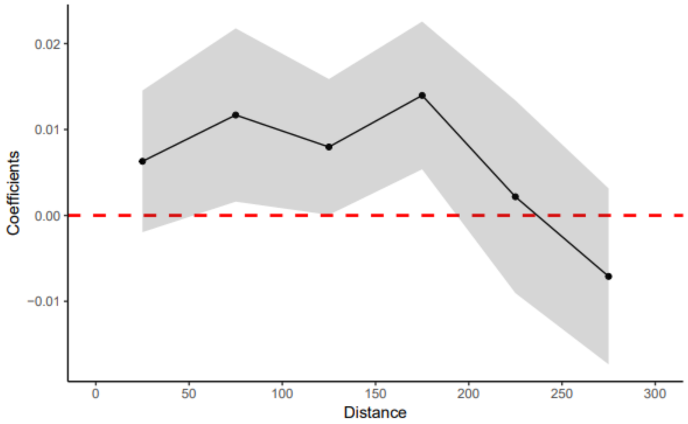
Notes: The graph plots the fixed effects coefficients of the Leads and Lags dummies using the set of controls from our preferred specification (Table 1, Column 3). The shaded area is a 95% confidence interval.
Data Sources: MapBiomas, MMA and IBGE

Based on this graph, I can assume that the parallel pre-trends hypothesis cannot be rejected in any year before the policy and that there is a persistent and increasing impact of the policy in the following years. This result is consistent with the coefficients of Table 1 and also with the fact that more municipalities were added to the list in 2009, 2011, and 2012. In summary, the evidence supports the claim that both groups have similar trends in the absence of the treatment.

6.2 Treatment Cut-off Robustness Check

As discussed in Section 5.2, it is necessary to check if the results are being driven by an arbitrary choice of treatment cut-off and also to be sure that the control group is not being affected by the policy. To address both concerns, I use the same model (1), though add a sixth *Treat_break* dummy for the 250-300 km distance interval instead of classifying the control group, like the ones more than 250 km far away I consider using a 300 km threshold.

Figure 5: Distance Breaks Robustness Coefficients - 300 km



Notes: The graph plots the fixed effects coefficients from the robustness specification similar to the one used in (Table 1, column 3) but adding an extra regressor: After x Treat (250– 300 km). In this case, the control group are the municipalities more than 300 km far away from the closest PM (instead of 250 km as in the baseline specification

Figure 5 shows results similar to the ones in Figure 2. The coefficients for intermediate distances (50-200 km) are significant at the 5% level and have similar magnitudes. The reach of displacement is 200 km because neither coefficient for (200-250 km and 250-300 km) is statistically significant. The (0-50 km) coefficient became non-significant. However, as explained before, the closest interval might have some deterrence effect mixing the results. It is also important to notice that this specification might suffer from a lack of statistical power, because when I changed the cut-off I made the proportion of units in the treatment group to be more distant from the optimal ratio of equal distribution, thus there is less variance on the regressors and less precision for statistical inference. Briefly, it can be concluded that

the overall results did not change much. Therefore, they are robust to the cut-off definition.

6.3 Caveats of the Model

Although this model seems to identify a causal spatial spillover impact of the policy, it can still suffer from omitted variable bias and possible biases caused by spatial autocorrelation in the dependent variable. For example, if unobserved policies were implemented after 2008, in the control group and not in treatment it might make us overestimate the impact, but since I am restricting the sample to the same administrative region Legal Amazon, it minimizes the probability of these biases.

7 Final Considerations

This research provides evidence of cross-biome leakage across the Amazon and the Cerrado borders. Leakage can make it more difficult to achieve national reductions in emissions. It fills the gap in the anti-deforestation evaluation literature by focusing on a less explored region, Cerrado, and less explored impact, spillovers. This paper also contributes to the Crime Literature by estimating spillover effects generated by a hotspot policing strategy.

The results suggest that the blacklist policy generated a displacement effect, mostly at intermediate distances (50-200 km). I use a differences-in-differences strategy to establish a causal relationship between the policy of interest and the side-effect generated by it. Robustness tests provide supporting evidence for the parallel trends assumption, and the coefficients are stable when controls are gradually included and when the treatment definition changes. Moreover, I assess the economic relevance using a counterfactual simulation, estimating a scenario with no blacklist policy. From this exercise, I estimate a leakage of 4,963 km^2, representing an offset of 29% of direct impacts.

In the light of these results, I argue that is necessary to extend existing policies, like DETER, to be able to detect clearings in other vegetation and allow the government to issue more alerts in the Cerrado. Also, it is necessary to make more specific policies that take into account biome differences and attempt to reconcile agricultural production and conservation efforts. Lastly, this paper and previous works (Gonzalez-Navarro, 2013; Davis, 2008; Andrade, 2016; Pfaff and Robalino, 2017; Gandour, 2018) have shown spillover effects to be relevant, so these effects always need to be considered in Policy Impact Evaluations.

For future research, as pointed out by Andrade (2016), land use and forest loss variables may be spatially dependent. Therefore, a spatial econometric model should be used to correct for this spatial autocorrelation. Furthermore, spatial heterogeneities might be better captured when using data in a more finer-scale than the municipality level (Donaldson and Storeygard, 2016). Finally, it would be useful to include in a single analysis of the spillover effects in both biomes to calculate the net impact on neighbors.

8 References

Abman, Ryan. 2015. "Reelection Incentives, Blacklisting and Deforestation in Brazil." San Diego State University, Department of Economics working paper.

Amin, Ariane Manuela, Johanna Choumert, Jean-Louis Combes, Pascale Combes Motel, Eric Nazindigouba Kere, Jean Galbert Ongono Olinga, and Sonia Schwartz. 2017. "A spatial econometric approach to spillover effects between protected areas and deforestation in the Brazilian Amazon." 27.

Andrade, Luiza Cardoso de. 2017. "Spillover effects of blacklisting policy in the Brazilian Amazon." PhD diss., Universidade de São Paulo, 2017.

Aragao, Luiz Eduardo OC, Yadvinder Malhi, Nicolas Barbier, Andre Lima, Yosio Shimabukuro, Liana Anderson, and Sassan Saatchi. 2008. "Interactions between rainfall, deforestation and fires during recent years in the Brazilian Amazonia." Philosophical Transactions of the Royal Society B: Biological Sciences 363, no. 1498: 1779.

Arima, Eugenio Y., Paulo Barreto, Elis Araújo, and Britaldo Soares-Filho. 2014. "Public policies can reduce tropical deforestation: Lessons and challenges from Brazil." Land use policy 41: 465-473.

Assunção, Juliano, and Romero Rocha. 2014. "Getting greener by going black: The priority municipalities in brazil." Rio de Janeiro, Brazil: Climate Policy Initiative.

Assunção, Juliano, Clarissa Gandour, and Rudi Rocha. 2015. "Deforestation slowdown in the Brazilian Amazon: prices or policies?" Environment and Development Economics 20, no.: 697-722.

Assunção, Juliano, Clarissa Gandour, and Romero Rocha. 2017. "DETERing deforestation in the Amazon: Environmental monitoring and law enforcement." Working Paper, Climate Policy Initiative, Núcleo de Avaliaçao de Polıticas Climáticas, Pontifica Universidade Católica (PUC), Rio de Janeiro: 36.

Barr, Robert, and Ken Pease. 1990. "Crime placement, displacement, and deflection." Crime and justice 12: 277-318.

Bertrand, Marianne, Esther Duflo, and Sendhil Mullainathan. 2004. "How much should we trust differences-in-differences estimates?." The Quarterly journal of economics 119, no. 1: 249-275.

Braga, Anthony A., David M. Hureau, and Andrew V. Papachristos. 2011. "The relevance of micro places to citywide robbery trends: a longitudinal analysis of robbery incidents at street corners and block faces in Boston." Journal of Research in Crime and Delinquency 48, no. 1: 7-32.

Brasil. 2007. "Decreto 6.321/2007." Brasília, DF, Brazil.

Brasil. 2008. "Decreto 6.514/2008." Brasília, DF, Brazil.

Brasil. 2012. "Código Florestal (Lei 12.651/2012)." Brasília, DF, Brazil.

Burgess, Robin, Matthew Hansen, Benjamin A. Olken, Peter Potapov, and Stefanie Sieber. 2012. "The political economy of deforestation in the tropics." The Quarterly Journal of Economics 127, no. 4: 1707-1754.

Cardoso Da Silva, José Maria, and John M. Bates. 2002. "Biogeographic Patterns and Conservation in the South American Cerrado: A Tropical Savanna Hotspot: The Cerrado, which includes both forest and savanna habitats, is the second largest South American biome, and among the most threatened on the continent." AIBS Bulletin 52, no. 3: 225-234.

Casa Civil. 2004. "Plano de Ação para a Prevenção e Controle do Desmatamento na Amazônia Legal. Operational plan, Grupo Permanente de Trabalho Interministerial para a Redução dos Índices de Desmatamento da Amazônia Legal." Casa Civil, Presidência da República, Brasília, DF, Brazil.

Chalfin, Aaron, and Justin McCrary. 2017. "Criminal deterrence: A review of the literature." Journal of Economic Literature 55, no. 1: 5-48.

Cisneros, Elías, Sophie Lian Zhou, and Jan Börner. 2015. "Naming and shaming for conservation: evidence from the Brazilian Amazon." PloS one 10, no. 9: e0136402.

Clarke, Ronald V., and David Weisburd. 1994. "Diffusion

of crime control benefits: Observations on the reverse of displacement." Crime prevention studies 2: 165-184.
Clarke, Ronald V. 1995. "Situational crime prevention." Crime and justice 19: 91-150.
Davis, Lucas W. 2008. "The effect of driving restrictions on air quality in Mexico City." Journal of Political Economy 116, no. 1: 38-81.
Donaldson, Dave, and Adam Storeygard. 2016. "The view from above: Applications of satellite data in economics." Journal of Economic Perspectives 30, no. 4: 171-98.
Eck, John E. 1993. "The threat of crime displacement." In Criminal Justice Abstracts, vol. 25, no. 3, pp. 527-546.
Gabor, Thomas. 1990. "Preventing crime: Current issues and debates." Canadian Journal of Criminology 32, no. 1.
Gandour, Clarissa. 2018. "Forest Wars: A Trilogy on Combating Deforestation in the Brazilian Amazon". PhD diss., Pontifica Universidade Católica (PUC).
Greenstone, Michael, and Rema Hanna. 2014. "Environmental regulations, air and water pollution, and infant mortality in India." American Economic Review 104, no. 10: 3038-72.
Gonzalez-Navarro, Marco. 2013. "Deterrence and geographical externalities in auto theft." American Economic Journal: Applied Economics 5, no. 4: 92-110.
Hargrave, Jorge, and Krisztina Kis-Katos. 2013. "Economic causes of deforestation in the Brazilian Amazon: a panel data analysis for the 2000s." Environmental and Resource Economics 54, no. 4: 471-494.
Hesseling, Rene. 1994. "Displacement: A review of the empirical literature." Crime prevention studies 3, no. 1: 97-230.
IBGE. 2004. "Mapa de Biomas do Brasil, primeira aproximação." Rio de Janeiro: IBGE 1.
IPCC. 2007. "Climate Change 2007". Synthesis report, Intergovernmental Panel on Climate Change.
Klink, Carlos A., and Ricardo B. Machado. 2005. "Conservation of the Brazilian cerrado." Conservation biology 19, no. 3: 707-713.

Matsuura, K., and C. J. Willmott. 2015. "Terrestrial air temperature: 1900–2008 gridded monthly time series (version 4.01), University of Delaware Dept. of Geography Center.".
MMA. Ministério do Meio Ambiente. 2011. "Monitoramento do Desmatamento nos Biomas Brasileiros por Satélite. Acordo de Cooperação Técnica MMA/IBAMA: Monitoramento do Bioma Caatinga 2008 a 2009."
Myers, Norman, Russell A. Mittermeier, Cristina G. Mittermeier, Gustavo AB Da Fonseca, and Jennifer Kent. 2000. "Biodiversity hotspots for conservation priorities." Nature 403, no. 6772: 853.
Nobre, Antonio Donato. 2014. "O futuro climático da Amazônia." Relatório de avaliação científica.
Nobre, Carlos A., Piers J. Sellers, and Jagadish Shukla. 1991. "Amazonian deforestation and regional climate change." Journal of climate 4, no. 10: 957-988.
Noojipady, Praveen, C. Douglas Morton, N. Marcia Macedo, C. Daniel Victoria, Chengquan Huang, K. Holly Gibbs, and L. Edson Bolfe. 2017. "Forest carbon emissions from cropland expansion in the Brazilian Cerrado biome." Environmental Research Letters 12, no. 2: 025004.
Pfaff, Alexander, and Juan Robalino. 2017. "Spillovers from conservation programs." Annual Review of Resource Economics 9: 299-315.
Short, Martin B., P. Jeffrey Brantingham, Andrea L. Bertozzi, and George E. Tita. 2010. "Dissipation and displacement of hotspots in reaction-diffusion models of crime." Proceedings of the National Academy of Sciences.
Valeriano, Dalton M., Eliana MK Mello, José Carlos Moreira, Yosio E. Shimabukuro, Valdete Duarte, I. M. Souza, J. R. Santos, Claudio CF Barbosa, and R. C. M. Souza. 2004. "Monitoring tropical forest from space: the PRODES digital project." International Archives of Photogrammetry Remote Sensing and Spatial Information Sciences 35: 272-274.
Weisburd, David, Laura A. Wyckoff, Justin Ready, John E.

Eck, Joshua C. Hinkle, and Frank Gajewski. 2006. "Does crime just move around the corner? A controlled study of spatial displacement and diffusion of crime control benefits." Criminology 44, no. 3: 549-592.

9 Appendix

9.1 *Data - Variable Constructions and Sources*

To define the biomes boundaries I used the one made available by MapBiomas, based on Biomes Limits Map from IBGE and refined using the Territories Limits and the phytophysiognomies Map. For the municipalities boundaries, I used the 2015 IBGE definition.

9.1.1 *Farming*

MapBiomas is a multi-institutional initiative that since 2015 aims to generate a time-series with annual data of the land cover and land use for all Brazilian biomes. The initiative automatizes the satellite images with advanced techniques like Random Forest. So, to obtain a good proxy of deforestation, I aggregated the categories relative to farming use (pasture and agriculture) to the municipality level, using the collection 2.3, available in the raster format with 30 meters resolution. After aggregating the area of interest, I divided it by the municipality area that is inside the Legal Amazon and the Cerrado biome, generating the fraction of the municipal area destined to farming.

9.1.2 *Treatment - Distance Criteria*

To define the treatment region, I first calculated the distance to the closest Priority Municipality of the 2008 list, then generated indicator variables based on 50 km distance breaks. The breaks used were: 0-50 km, 50-100km, 100-150 km, 150-200 km, 200-250 km, and 250-300 km. In the baseline, I include all of these five dummies, so the control group is the omitted category (i.e., all municipalities above 300 km). To calculate the distances, I spatialized the PM list provided by the MMA based on the 2015 IBGE municipalities division.

9.1.3 *Agricultural Commodity Prices*

The source for the annual price index (USD, 2010 base year) was the World Bank Pink Sheet. I gathered data for soybean, rice, sugar (as a proxy for sugarcane), and cattle. To add variance across the municipalities, I weigh the prices by the commodity relevance in each municipality. For that I used data from IBGE on agricultural production and the following formula:

$$PPA_{itc} = PP_{tc} \times A_{ic,2000-2003}$$

where PPA_{itc} is the weighted real price of commodity c in municipality i and year t; PP_{tc} is the Pink Sheet real price of commodity c in year t' and A_{ic},2000–2003 is the municipality specific weight. For crops, the weight is given by the share of the municipal area used as farmland for crop c in municipality i averaged over 2000 and 2003. To avoid endogeneity, I only consider the period before the sample for the analysis and the policies implementation. For beef cattle, given that annual pasture specific for beef is unobservable, the weight is given by the ratio of heads of cattle to the municipal area in municipality i averaged over 2000 and 2003.

9.1.4 Weather Control

I compiled weather data from the Matsuura and Willmott (2015) dataset that created a regular grid worldwide of estimated precipitation and temperature over land. They use extrapolations techniques based on data collected at weather stations. It is a monthly dataset, so for precipitation, I calculated a total value by year and for temperature and took the annual average.

9.1.5 Policy Control

For the Cerrado Priority Municipalities, I did the same spatializing process as the Amazon Priority Municipalities and then created an indicator variable equal to 1 if the municipality i in year t was in the list. For the protected areas, I gathered data from multiple sources (FUNAI, ISA and MMA) and calculated the fraction of the municipal area that was legally protected for each sample year.

9.2 Summary Statistics

Table 3: Summary Statistics Table

	2004	2005	2006	2007	2008
Farming	0.373	0.373	0.374	0.378	0.379
	(0.195)	(0.193)	(0.192)	(0.192)	(0.193)
Distance	168.5	168.5	168.5	168.5	168.5
	(123.2)	(123.2)	(123.2)	(123.2)	(123.2)
After	0	0	0	0	0
Treatment (0-50 km)	0.180	0.180	0.180	0.180	0.180
Treatment (50-100 km)	0.130	0.130	0.130	0.130	0.130
Treatment (100-150 km)	0.169	0.169	0.169	0.169	0.169
Treatment (150-200 km)	0.175	0.175	0.175	0.175	0.175
Treatment (200-250 km)	0.118	0.118	0.118	0.118	0.118
Treatment (250-300 km)	0.0873	0.0873	0.0873	0.0873	0.0873
Control (>250km)	0.228	0.228	0.228	0.228	0.228
Control (>300km)	0.141	0.141	0.141	0.141	0.141
Price, Corn	1.419	1.214	1.462	1.851	2.341
	(2.700)	(2.310)	(2.782)	(3.522)	(4.455)
Price, Sugarcane	0.000333	0.000445	0.000649	0.000417	0.000491
	(0.00188)	(0.00252)	(0.00367)	(0.00236)	(0.00278)
Price, Soybean	8.646	7.512	7.165	9.653	12.19
	(24.89)	(21.62)	(20.63)	(27.79)	(35.10)
Price, Rice	3.051	3.563	3.701	3.734	6.902
	(4.074)	(4.758)	(4.942)	(4.986)	(9.216)
Price, Cattle	91.20	92.09	87.42	84.18	94.18
	(77.55)	(78.31)	(74.34)	(71.58)	(80.08)
Rain	331.2	278.6	327.3	278.9	323.0
	(211.9)	(192.0)	(211.7)	(189.3)	(206.8)
Temperature	25.65	25.89	25.65	25.99	25.45
	(1.345)	(1.488)	(1.275)	(1.303)	(1.368)
Cerrado Priority Muni	0	0	0	0	0.0310
Protected Area	0.110	0.110	0.112	0.112	0.112
	(0.239)	(0.239)	(0.239)	(0.239)	(0.239)
Observations	355	355	355	355	355

Note: The table reports annual averages and standard deviations (in parenthesis) at the municipal level for the variables used in the analysis. The sample includes all Legal Amazon Municipalities of the Cerrado Biome. Sources and units: Farming (share of the municipal area destined for Farming, MapBiomas); Distance (distance in kilometers to the closest Pirority Muni (2008), IBGE and MMA); After 1{year > 2008}; Treatment (break km) 1{Distance ≤ break}; Control (>x km) 1{Distance > xkm}; Prices (year 2010 USD, World Bank, PAM/IBGE and PPM/IBGE); Rain (annual average millimeters, Matsuura e Willmott (2015)); Temperature (annual average celsius degrees, Matsuura e Willmott (2015)); Cerrado Priority Muni (MMA), Protected Area (share of the municipal area that is protected, INCRA *and FUNAI). The table was divided into two parts 2004-2008 and 2009-2014. Standard deviations were omitted for dummy variables.*

Table 4: Summary Statistics Table 2009-2014

	2009	2010	2011	2012	2013	2014
Farming	0.381	0.383	0.387	0.393	0.396	0.396
	(0.193)	(0.195)	(0.197)	(0.196)	(0.195)	(0.195)
Distance	168.5	168.5	168.5	168.5	168.5	168.5
	(123.2)	(123.2)	(123.2)	(123.2)	(123.2)	(123.2)
After	1	1	1	1	1	1
Treatment (0-50 km)	0.180	0.180	0.180	0.180	0.180	0.180
Treatment (50-100 km)	0.130	0.130	0.130	0.130	0.130	0.130
Treatment (100-150 km)	0.169	0.169	0.169	0.169	0.169	0.169
Treatment (150-200 km)	0.175	0.175	0.175	0.175	0.175	0.175
Treatment (200-250 km)	0.118	0.118	0.118	0.118	0.118	0.118
Treatment (250-300 km)	0.0873	0.0873	0.0873	0.0873	0.0873	0.0873
Control (>250km)	0.228	0.228	0.228	0.228	0.228	0.228
Control (>300km)	0.141	0.141	0.141	0.141	0.141	0.141
Price, Corn	1.852	2.006	2.836	2.923	2.551	1.927
	(3.523)	(3.818)	(5.397)	(5.561)	(4.854)	(3.666)
Price, Sugarcane	0.000743	0.000840	0.000925	0.000772	0.000637	0.000622
	(0.00420)	(0.00475)	(0.00523)	(0.00437)	(0.00360)	(0.00352)
Price, Soybean	10.86	10.79	11.69	12.87	11.77	10.92
	(31.27)	(31.06)	(33.64)	(37.06)	(33.88)	(31.43)
Price, Rice	6.280	5.337	5.341	5.577	5.033	4.273
	(8.386)	(7.126)	(7.132)	(7.447)	(6.721)	(5.705)
Price, Cattle	84.35	103.4	112.4	116.0	114.6	141.4
	(71.73)	(87.95)	(95.58)	(98.65)	(97.43)	(120.2)
Rain	353.0	293.5	352.6	283.3	320.1	327.6
	(220.5)	(202.8)	(231.2)	(201.1)	(216.4)	(220.7)
Temperature	25.41	26.28	25.50	25.58	25.60	25.53
	(1.262)	(1.515)	(1.320)	(1.478)	(1.584)	(1.325)
Cerrado Priority Muni	0.0338	0.0338	0.0423	0.113	0.113	0.113
Protected Area	0.114	0.114	0.114	0.114	0.114	0.114
	(0.242)	(0.242)	(0.242)	(0.242)	(0.242)	(0.242)
Observations	355	355	355	355	355	355

Note: The table reports annual averages and standard deviations (in parenthesis) at the municipal level for the variables used in the analysis. The sample includes all Legal Amazon Municipalities of the Cerrado Biome. Sources and units: Farming (share of the municipal area destined for Farming, MapBiomas); Distance (distance in kilometers to the closest Priority Muni (2008), IBGE and MMA); After 1{year > 2008}; Treatment (break km) 1{Distance □ break}; Control (>x km) 1{Distance > xkm}; Prices (year 2010 USD, World Bank, PAM/IBGE and PPM/IBGE); Rain (annual average millimeters, Matsuura e Willmott (2015)); Temperature (annual average celsius degrees, Matsuura e Willmott (2015)); Cerrado Priority Muni (MMA), Protected Area (share of the municipal area that is protected, INCRA and FUNAI). The table was divided into two parts 2004-2008 and 2009-2014. Standard deviations were omitted for dummy variables.

[THIS PAGE INTENTIONALLY LEFT BLANK]

Intertemporal Relationships between Bid Price and Number of Bids: Evidence from Singapore's Vehicle Quota System

Luther Yap

University of Cambridge

Abstract

In Singapore's Vehicle Quota System (VQS), individuals have to successfully bid for a Certificate of Entitlement (COE) to own a vehicle. Based on the type of vehicle, a different category of COE is required. This paper focuses on the relationship between the bid price and number of bids of Category A (cars with low engine capacity and power), Category B (cars with high engine capacity and power) and Category E (unrestricted) COE, which are imperfect substitutes for a potential car buyer. An intertemporal micro-founded model is used to show how prices and number of excess bids are simultaneously determined and depended on expectations that are formed from history. Structural Vector Autoregressive (SVAR) models are then used to show how prices and number of bids of various COE categories are related intertemporally and contemporaneously.

Table of Contents

1 Introduction

In light of the scarcity of land in Singapore and its congested roads, a Vehicle Quota System (VQS) was implemented in May 1990 and has since been used to control the quantity of vehicles on the roads. In the VQS, potential car owners must bid for a Certificate of Entitlement (COE), and contingent on their success, these individuals have the right to own the vehicle for ten years. Since its implementation, the VQS has undergone several changes, including transferability, categorization of COE types, frequency of COE auctions, and the auction format used. Currently, COE auctions are held semi-monthly using a second price open bid system.

Singapore's VQS acts as a natural experiment in second price auctions that has generated much academic interest locally. Koh and Lee (1993) exploited the policy change in October 1991 from a transferable COE system to a non-transferable COE system to investigate whether auctions with resale markets result in higher prices. Koh et al. (2007) investigated the change in bidding behavior when the COE auction format changed from a sealed-bid to an open-bid system in July 2001 and found that prices in open-bid formats were lower. Beyond empirical outcomes of auctions, most literature on Singapore's VQS study microeconomic outcomes including social diversity in vehicle ownership (Chu, 2012), market structure of Singapore's car distributorship industry (Koh, 2003), impact on housing prices (Huang et al., 2018), and impact on motorcycle population (Chu, 2018).

The existing COE literature, however, has yet to explicitly address the relationship between the price of COE and the number of bids in each session, as well as their intertemporal relationships, which is the gap this paper aims to fill. The relationship between bid price and the number of bidders has been previously studied: Hungria-Gunnelin et al. (2013) analysed real estate auctions in Stockholm, Sweden, and found that extra bids increase the average price per square meter. Additionally, Han´ak and Muchov´a (2015) found that increasing competition (i.e. the number of competing firms)

affects the award price of public works contracts. With such empirical evidence, and as would be expected intuitively, it is hypothesized that a larger number of bids at a COE session will increase the price.

This paper deepens our understanding of the relationship between bidders and price by considering two further aspects of the problem: (i) bidders who fail to obtain a COE in one period can try again in the next period, so intertemporal decision-making should be considered, which makes the analysis more complex than one-off second price auction theory problems (ii) the good that bidders are bidding for have close substitutes in other categories, so trading off their relative valuation should be considered. This paper aims to model these intertemporal and substitution aspects and test them empirically. The systematic study of the COE game supported by its time series in recent years is this paper's main contribution to literature.

Within the literature on dynamic auctions, first-price sealed bids have been investigated (Jofre-Bonet and Pesendorfer, 2000), and there are some theoretical studies on the dynamic Vickrey auctions with particular specifications (Mierendorff, 2013). This study extends this field of literature by utilizing the natural experiment with the COE to verify theoretical predictions of dynamic Vickrey auctions. Besides theoretical interest, there is also some commercial value to studying COE price movements because car dealers rely heavily on accurate price expectations of COE to maximize profits.

To analyze intertemporal aspects of the price-bidder relationship in COE, this paper will develop a simple micro-founded theoretical model for how prices and the number of bidders are determined, then estimate a Structural Vector Autoregressive (SVAR) model to test and to verify the theoretical propositions that prices and bids are related contemporaneously and intertemporally. This paper will test for the joint significance of lags and appeal to Granger causality. Main findings are summarized in Results 1 and 2.

The rest of the paper will proceed as follows. Section 2 discusses in detail the institutional background, including how the VQS works presently, the dataset used, and existing time trends. Section 3 develops the theoretical model for intertemporal choice and discrete choice to show how price and the number of bidders may be related. Using the theoretical result, Section 4 discusses an empirical strategy for estimation using the COE data. Section 5 discusses the results for the VAR estimates and its subsequent robustness checks, and Section 6 concludes.

2 Background

The current VQS uses an electronic second price open bid auction format with five bid categories. In each category, there is a predetermined quota of COEs available, say k. At the end of a bid session, the top k bids will receive the COE, and all k bidders will pay the price of the $k + 1^{th}$ highest bid plus \$1 for the COE (i.e. the highest unsuccessful bid plus a dollar) - this price is known as the Quota Premium (QP). The auction is open in that at any time during the bidding session, all bidders will observe the clearing QP in that moment. In other words, should there be no further action from the instant of viewing the QP till the end of the session, that particular price will be the session QP. I will call this instantaneous clearing QP the Intermediate Price (IP). Consequently, the session QP should be at least as high as the IP displayed during the bidding session, and all bidders who placed bids above the session QP will obtain the COE. It is possible, but costly, for bidders to change their bids during the session, and individuals are not allowed to place more than one bid. If the number of bids is less than the quota, then the QP will be \$1. The five COE bid categories are as follows:[1]

Category A: Car up to 1600cc & 97kW
Category B: Car above 1600cc or 97kW
Category C: Goods Vehicle and Bus
Category D Motorcycle
Category E: Open - all except motorcycle

These five categories have existed in the system since May 1999, although a refinement was made in September 2013 where the condition of 97kW (which did not previously exist) was included in Category A such that the car models with capacity of less than 1600cc but above 97kW moved from Category A to Category B.

This paper uses publicly available data from April 2002 to July

1 Detailed information on the VQS can be found on publications by the Land Transport Authority (LTA), including this: https://www.lta.gov.sg/content/dam/ltaweb/corp/ocoe.pdf

2018. [2] The dataset only contains information on the number of bids, the QP for each session, and the quota. While information on the distribution of bids is unavailable, existing information would suffice for the purpose of this study. The current system of having two bidding sessions a month began in April 2002 - bidding was done once a month prior to April 2002. Thus, the only possible discontinuity should occur in September 2013 due to the refinement in Category A.

The time trends of four key variables can be observed in Figure 1 in the appendix. Panel A shows the time trend for the QP of all five categories, which will henceforth be called the *price* variable. Both Category C and D have prolonged periods where the price is exceptionally low, and some seasons when the number of bids is less than the quota. It is unsurprising that the price of Category E is typically one of the highest in each session. Should the price of E be lower than, say, B, bidders can simply pay a lower price for E and still drive their Category B cars, so E should be among the highest by arbitrage. A and B may also be regarded as close substitutes, as the price of a COE is expected to influence the type of car that a potential car buyer would like to purchase, as will be expounded on in Chapter 3. As expected in a speculative market, the time series look nonstationary. Panel B reports the trends in COE quota, henceforth denoted as the *quantity* variable. This is determined by the Land Transport Authority's (LTA) vehicle growth target for a time period, smoothed out over several bidding sessions. Consequently, one would expect some flat portions in the trend.

Since the more interesting analysis arises from Categories A, B and E, only their time trends are reported in Panels C and D. Panel C shows the number of bids in each session, henceforth denoted as *bids*. Panel D shows the time trends of excess bids, where *excess* = *bids*–*quantity*. This will be meaningful because price is affected by bids only when bids > quantity (i.e. excess) > 0. With quantity being exogenously determined, the task of investigating the relationship between the number of bids and price can be reduced to investigating the relationship between *excess* and *price*.

2 Dataset can be found here: https://coe.sgcharts.com/

3 Theoretical Model

This paper uses a micro-founded intertemporal model to explain the interaction between the price of a COE in each category and the number of bidders. The model will proceed in three parts: (1) a simple two-period intertemporal model, (2) a discrete choice problem between COEs of two substitutable categories, and (3) the interaction of the first two parts that would allow the simultaneous determination of price and the number of bidders. The last two subsections explain how the model can be empirically tractable by using historical prices and number of bids, and quantify the nature of substitution between COE categories.

3.1 Intertemporal Model

For simplicity, we first consider an economic agent who wants to obtain a COE for a specific category. An individual can choose to bid for the COE in the current period or in the next period. At the point of choosing whether to bid, they do not yet know the price. Their expected utility (U) in each period is dependent on his valuation (V) of the COE and the expected price ($E(P)$). Formally, using subscript i to denote individual i, utility in the current period t is $U_{i,t} = V_i - E(P_t)$ and utility in the next period is $U_{i,t+1} = V_i - E(P_{t+1})$.

As is the practice of most intertemporal models in economics since Samuelson (1937), a discount factor will be used. With β as the discount factor, the rational agent will bid in the current period if and only if $U_{i,t} \geq \beta U_{i,t+1}$. By substitution, we will obtain $V_i - E(P_t) \geq \beta(V_i - E(P_{t+1}))$. Making V the subject of the formula, the condition for bidding is:

$$V_i \geq \frac{1}{1-\beta_i}[E(P_t) - \beta_i E(P_{t+1})] \tag{1}$$

Here we allow heterogeneity across individuals in terms of valuation of COEs and discount factors. When the value of the right-hand side increases, condition (1) is more likely to fail, so we can intuitively observe how various factors may affect the decision to bid. Bidding in the current period (t) is less likely when the individual is more patient (higher βi), expects the current price to be higher, or when they expect the future price to be lower. This model can easily be generalised, but we shall keep the two-period model for now so that it can be combined with the discrete choice model more easily.

This model can be extended to bulk bidding by car dealerships. Empirically, bidding is mostly done by car dealerships as they are able to spread the risk across many bids: a tactic beyond the abilities of individual bidders. In the current non-transferable COE system, the car dealership must obtain an order from a customer before it can bid for a COE. There will be a time discount factor because of the urgency in delivering the car to the customer to meet sales targets. Instead of an intrinsic valuation of the COE, *V* would translate to the revenue that the company can potentially obtain by selling the package that includes both the car and the COE. In light of differing car valuations, car dealerships actively choose the number of bids to place in each period, so this model is analogously applicable. However, it should be noted that when a firm has more than one bid, its strategy will be different from that of individuals because bidding V_i is no longer a weakly dominant strategy when its other bids can influence the price.

3.2 *Discrete Choice Model*

If an individual wishes to purchase a car and bid for a COE in a given time period, they still have to choose the type of car they wish to buy. The individual then indirectly chooses the type of COE that they will bid for - Category A or B. Using a similar setup, where utility is given by the difference between the valuation and the price paid for the COE, the expected utility of A and B will be as follows:

$$U_{A,i} = V_{A,i} - E(P_A)$$
$$U_{B,i} = V_{B,i} - E(P_B)$$

Using the condition that the individual chooses to bid for A

if and only if $U_{A,i} \geq U_{B,i}$, the condition for bidding for A will be given in Equation 2 below.

$$V_{A,i} - E(P_A) \geq V_{B,i} - E(P_B) \tag{2}$$

3.3 *Price and Number Determination*

Combining the two models above, we know that an individual will bid for a category A COE in period *t* if and only if criteria (1) and (2) are both fulfilled. Thus, when there are N potential bids in the economy, the number of bids will be given by Equation 3. Note that 1(.) is an indicator function that takes the value 1 if the condition is fulfilled, and zero otherwise.

$$bids_{A,t} = \sum_{i=1}^{N}[1(V_{A,i} \geq \frac{1}{1-\beta_i}[E(P_{A,t}) - \beta_i E(P_{A,t+1})]) \times 1(V_{A,i} - E(P_{A,t}) \geq V_{B,i} - E(P_{B,t}))] \tag{3}$$

I will assume that price expectation is a function of the session's prices. During the session, all bidders observe the intermediate clearing QP (i.e. IP), so $E(P_t)$ will be revised based on the instantaneous IP. Thus, expectations will be a function of IP, which eventually converges to QP.

Assumption 1. *Price expectation is a function of the intermediate price, among others. i.e.* $E(P_t) = f(IP_t, \cdots)$.

From Equation 3, we have a theoretical result for the number of bids for a particular category in a particular bidding session. This is a function of price expectations (intertemporally and between different categories), relative valuation, and the discount factor. From Assumption 1, these price expectations are also a function of IP, so bids are a function of IP. Thus, the remaining task is to determine the price.

Bid prices will be drawn from the following interval: $bidprice_i \in [IP, V_i]$, where the IP is the latest seen price during the bidding session. Bidding less than the IP is irrational for any individual or company as it is essentially an ineffective bid, as the price will definitely be at least as high as the IP. Bidding more than *V* is also irrational as it may result in negative utility. Since the next bidder entering the auction must bid at least the IP, the clearing price must be weakly increasing with the number of bidders during any given bidding session. Since the price can exceed $1 if and only if the number of bids exceeds the quota, it is more meaningful to look at *excess* = *bids–quantity*.

Since P_t is the outcome of individual price decisions, it must be a function of price expectations. These expectations are formed by all existing information. I denote this information set as Ω. Allowing heterogeneity in valuation, this is encapsulated in the following equation.

$$P_{A,t} = f(\{E(P_{A,t}|\Omega)\}_{i \in n}, \{V_{A,i}\}_{i \in n}, excess_{A,t}) \qquad (4)$$

Using Equation 3 and Equation 4, we should expect simultaneity between price and excess of any COE category. Excess is affected by the current period price in the open bid system: during the session, bidders already have some (although imperfect) information about P_A due to the last seen price that would shape $E(P_A)$, which would influence the number of bidders. We have also established that $excess_A$ affects $price_A$ directly from the way that prices are drawn from the distribution.

Proposition 1. *The contemporaneous price and excess number of bids for a given COE category are simultaneously determined.*

Proof. From Equation 4, price is a function of excess. From Assumption 1, bids is a function of IP. By definition, IP_t weakly increases and converges to P_t during the bid session i.e. $IP_t \to P_t$. Thus, bids are a function of P_t. By construction, excess = bids–quantity, so excess is a function of P_t. $\square$

3.4 Relevance of Price and Excess Histories

The model thus far has shown how price and excess are simultaneously determined and how both of them depend on price expectations, which are conditioned on the information set Ω. This subsection shows how Ω is composed of the history of price and excess.

Price expectations are conditioned on the prices in the previous periods: the price history of both some A and its substitute B will be relevant. The expectation of P_A should minimally depend on its previous lag. Theoretically, one should expect A and B to have similar trends (which is verified by the time series in Figure 1): if the prices of A and B diverge indefinitely, car owners would choose the category that is relatively cheaper, and thus push its price up again. In other words, prices of A and B are expected to move together, commensurate with their relative valuation. As such, $E(P_A)$ is partially determined by $E(P_B)$, which is also dependent on its own lag. Therefore, both price histories are relevant.

Price expectations also depend on previous period excess. If there is a large excess of bidders for A in the previous period, then there must be a large number of unsuccessful bidders who intended to obtain A. Since they wish to obtain A, we would expect them to bid again in the next period. This would increase the current period excess (by increasing N in Equation 3), and from Equation 4, would increase price in expectation. The same process would similarly occur for substitute B, so the expectation of $price_B$ would be determined by the history of $excess_B$. However, we already know that $E(P_B)$ partially determines $E(P_A)$, so excessB has some role in determining $E(P_A)$. The proof relies on the role of Ω.

Proposition 2. *For a given COE category, price and excess are in part determined by their own histories of price and excess.*

Proof. From Equation 4, price is a function of $E(P|\Omega)$ and from Equation 3, bids are a function of $E(P|\Omega)$. Information set Ω consists of its own history of price and excess. $\square$

Proposition 3. *Prices and excess of various COE categories are influenced by the prices and excess of their substitutes.*

Proof. From Equation 4, price is a function of $E(P|\Omega)$ and from Equation 3, bids are a function of $E(P|\Omega)$. Information set Ω consists of the history of price and excess of its substitutes. Contemporaneously, since $bids_{A,t}$ is a function of $E(P_{B,t})$, $E(P_{B,t})$ is a function of $IP_{B,t}$, and $IP_{B,t}$ □ $P_{B,t}$, $bids_{A,t}$ (and consequently $excess_A$) is a function of $P_{B,t}$. □

Intuitively, when observing a higher IP_A (or a rapidly increasing $excess_A$), one might expect $price_A$ to be higher, so they would choose to bid in substitute B instead. This would then affect priceB and excessB. A symmetric argument may be made the other way round. Thus, we expect contemporaneous simultaneity.

For ease of notation and explanation, the arguments in this section have denoted A as one COE category and B as its substitute, which plausibly corresponds to how Category A and Category B COEs are substitutes for each other. However, the model is more far-reaching in that it refers to any COE category and its substitute: the arguments can be analogously applied to the relationship between A and E COEs, between B and E COEs, and between A, B and E COEs.

3.5 Nature of Substitution

Thus far, the theoretical discussion has assumed that substitution between COE categories is symmetric, but this is not necessarily true. Consider first the substitution between A and E. Anyone who wishes to bid for A would also bid for E if the expected price of E were lower, but one who is bidding for E need not be able to bid for A (for instance, if he wishes to own a Category B car). The relationship between B and E is similarly asymmetric.

COE in categories A and B could either be imperfect substitutes or not substitutes at all. For a potential car buyer, A and B are imperfect substitutes: one could choose a Category B car and bid for B, or a Category A car and bid for A, with both

types of cars serving a similar function.

However, for a potential owner who is certain that they want to own a Category A car, A and B are not substitutable. A less obvious relationship between A and B concerns the nature of the car: Category B cars have engines of higher capacity, have more power, and tend to be more expensive. Thus, a consumer who is bidding for B is likely to have the ability to bid for A as well, but one bidding for A need not be able to bid for B.

It is beyond the scope of this paper to model the nature of such substitutions formally, but this intuition is useful when examining the gathered empirical results. Existing propositions are sufficient to formulate an empirical strategy, which is what the rest of this paper aims to do and analyse.

4 Empirical Strategy

In light of how price and excess are simultaneously determined and are dependent on histories, the most appropriate econometric model would be a Vector Autoregressive (VAR) model. Additionally, the model will be parsimonious, and will deal largely with price and excess time series, because overfitting the model may describe the history well, but is unreliable for forecasting. (Adhikari and Agrawal, 2013) This section discusses the preliminary tests conducted on the time series and justifies how the propositions can be tested with a SVAR(3) model.

A preliminary diagnostic assessment of time series would be to test its stationarity. To validate the robustness of the result, both the Augmented Dickey-Fuller (ADF) test and the Phillips–Perron (PP) test are used. The PP test is non-parametric and is robust to autocorrelation and heteroscedasticity. (Phillips and Perron, 1988) The ADF is sensitive to the number of lags used in the test: if there are too few lags, one ought to be worried about serial correlation possibly biasing the test results. If there are too many lags, one would lose power in rejecting the null of nonstationarity. The default number of Newey-West lags to use for the PP test is given by the integer part of $4(T/100)^{2/9}$, where T is the number of time periods, which translates to 5 lags in our time series. Similarly, 5 lags are used for the ADF test. The results of ADF and PP tests are reported in Table 1: ADF and PP are consistent in showing that all the price time series are non-stationary while all the excess time series are stationary. The ADF test is done without a time trend because no linear time trend is apparent when looking at the time series charts.

A further consideration of the specification would be whether to use levels or logs for prices. Taking the natural logarithm would help to linearize an exponential time trend, which is typical of many time series. However, by inspection, the time trend for prices warrants no such adjustment. Furthermore, there is a large drop in 2009 amplified by the logarithmic formulation. Inclusion of this observation would bias the estimates but the trimming of outliers would also compromise the model's forecasting ability. Since the problem of outliers can be circumvented by the levels formulation, the levels specification for prices is primarily used. To check the robustness of the result, the model is also estimated in log, and the results are largely similar.

Regressing stationary variables with nonstationary variables would lead to spurious results. To remedy this issue, the first differences are taken for all price series, denoted *dprice*. The first differences are unsurprisingly stationary using the ADF and PP tests above. As such, the final formulation of the model would involve some interaction of *dprice* and *excess* of some category A and its close substitute: possibly B or E.

Proposition 1 is ideally estimated using a simultaneous equation model. No adequate instrument may be found for *dprice*, but quantity is a good instrument for excess. The exclusion condition is satisfied because variation in quantity is determined exogenously by the Land Transport Authority (LTA), and it is impossible to affect *dprice* except through excess. Quantity is expected to be relevant to excess: as excess is a function of quantity. Since $excess = bids - quantity$, a positive coefficient on quantity in the system below would suggest that the number of bids responds more than one-for-one to any change in quantity. The equation system is as follows.

$$dprice_t = \beta_1 excess_t + a_t excess_t = \gamma_1 dprice_t + \gamma_2 quantity_t + b_t$$

The order condition is not satisfied in this system, but the *dprice* equation is still identifiable. Using 2SLS, $\hat{excess}_t$ can be obtained from regressing $excess_t$ on *quantity*. Then, regress $dprice_t$ on $\hat{excess}_t$ to obtain $\beta_{1,2SLS}$. Using the data for Category B, the first-stage F statistic is 76.35 > 10, so the relevance condition is satisfied. In the second stage, the test statistic on the β_1 coefficient is –1.17, so we do not reject the null of 0. [3]

Notably, the specification is incomplete because of other variables indicated in Proposition 2 and 3. However, this provides

3 Similar results are observed for Category A and E COE. Results are available upon request.

an initial diagnostic to suggest that excess may not contemporaneously affect *dprice* as we supposed it would in Proposition 1.

To test Propositions 2 and 3, a structural vector autoregression (SVAR) is estimated as it can incorporate both contemporaneous relationships between endogenous variables and intertemporal relationships. We are interested in estimating the structural equation in Equation 5, but only Equation 6 in its reduced form can be estimated. **A** and **B** are matrices of coefficients. With n endogenous variables, we consequently need n^2 restrictions. (Kilian, 2011) Using the approach from Sims (1980), I restrict the error matrix to be diagonal and use an upper triangular matrix for A. This is possible only with Assumptions 2 and 3.

$$\mathbf{A}\mathbf{y}_t = \mathbf{a} + \sum_j \mathbf{B}_j \mathbf{y}_{t-j} + \mathbf{u}_t \tag{5}$$

$$\mathbf{y}_t = \mathbf{A}^{-1}\mathbf{a} + \sum_j \mathbf{A}^{-1}\mathbf{B}_j \mathbf{y}_{t-j} + \mathbf{A}^{-1}\mathbf{u}_t \tag{6}$$

Assumption 2. *The dprice of a given COE category is unaffected by its contemporaneous excess.*

This assumption follows from the initial diagnostic. While Proposition 1 is rejected, it is still consistent to hold Propositions 2 and 3. Proposition 2 is unaffected because it relies on the information set, which is irrelevant to contemporaneous relationship. The proof of Proposition 3 similarly relies on the information set. As for contemporaneous relationships, price affecting excess is sufficient to prove that there are contemporaneous cross-relationships.

Assumption 3. *Variables of E do not contemporaneously affect A and B and variables of A do not contemporaneously affect B.*

This assumption relies on an intuitive argument for the nature of substitution. Between substitutable categories, an argument may be made for how restrictive these categories are. Suppose for simplicity that bidders attempt to bid for the more restrictive category first, and if the expected price of the restrictive category is too high, they switch to the less restrictive category. This is a more reasonable assumption than bidders going for the less restrictive category first if they are risk averse. Category E is the least restrictive, so a reasonable constraint would be for bidders not to switch from E to A and B, but to be able to switch from A and B to E. Thus, when $price_A$ and $price_B$ are too high, $excess_E$ may be contemporaneously affected, but the opposite is untrue. Arguably, Category B is more restrictive than A because B cars are more expensive, so one who is able to afford a B car is likely able to afford an A car. When the price of B is too high, the bidder might be willing to switch to A, but the other direction is less likely. Thus, the variables of A do not contemporaneously affect B.

To test Proposition 2, the vector of endogenous variables [4] is Assumption 2 is sufficient to make matrix A upper triangular for estimation.

$$\mathbf{y}_t = \begin{pmatrix} excess_t \\ dprice_t \end{pmatrix} \tag{7}$$

To test Proposition 3, both Assumptions 2 and 3 are required. All COE category pairs are estimated, but only the SVAR between B and E are reported as they are most substitutable. The vector of endogenous variables is

$$\mathbf{y} = \begin{pmatrix} excessE \\ dpriceE \\ excessB \\ dpriceB \end{pmatrix} \tag{8}$$

Finally, the SVAR with all three categories can be estimated with vector

$$\mathbf{y} = \begin{pmatrix} excessE \\ dpriceE \\ excessA \\ dpriceA \\ excessB \\ dpriceB \end{pmatrix} \tag{9}$$

4 The vectors of endogenous variables in Equations 7, 8 and 9 are distinct specifications of the SVAR model in Equation 5.

The propositions can be tested by appealing to Granger causality. Some variable X is said to Granger cause Y when lags of X are jointly significant in explaining Y: this can be tested using the Wald statistic. By running a VAR, one can exclude all lags of some variable X in a restricted model and test its difference from the unrestricted model with those lags included. If the difference is significant, then this variable X Granger causes Y, which helps us rigorously establish a relationship between Y and the lags of X. The remaining task, then, is to determine if X is positively or negatively associated with Y: this can be established by looking at the estimated coefficients in **A** and lags.

To determine the number of lags used in the VAR, various information criteria were calculated, including the Final Prediction Error (FPE), the Akaike Information Criterion (AIC), the Schwarz Bayesian Information Criterion (SBIC), and the Hannan and Quinn Information Criterion (HQIC). These statistics help us choose the optimal number of lags to be used; the optimal lag is the one that minimizes a given information criterion. For instance, the AIC is negatively related to the likelihood in the given model, so it is sensible to minimise the AIC. Equation 7 is optimally estimated with 2 lags, and Equations 8 and 9 are optimally estimated with 3 lags. Consequently, SVAR(2) and SVAR(3) are estimated for the respective specifications.

5 Results

This section will first present and interpret the findings from the SVAR models, then consider the sensitivity of results to differing specifications. In checking for robustness of the result, this paper first considers the logarithmic formulation, then partitions the data to before and after the policy change in September 2013.

5.1 SVAR of History and Category Pairs

The SVAR result for the model in Equation 7 estimated for category B is presented in Table 3. The estimates suggest that the effect of the lags of $excess_B$ on $dprice_B$ is ambiguous, but $excess_B$ is determined by $dprice_B$ - both contemporaneously and intertemporally. $excess_B$ is positively correlated with its own lags, which is intuitive because more excess in previous periods tends to result in more recurrent unsuccessful bidding. Interestingly, lags of $dprice_B$ negatively affects $excess_B$, but $dprice_B$ positively affects $excess_B$ contemporaneously. A historical increase in $price_B$ could lead deter potential bidders from entering the market, which decreases the number of bids, and hence $excess_B$. Contemporaneously, an increase in $dprice_B$ increases $excess_B$, which is rather counterintuitive by the theoretical account, but can be explained by risk aversion and higher anticipated future prices as the following period's price would adjust less than one-for-one to current period changes in price. Thus, when $dprice_B$ is increasing, more bidders bid for B for fear of higher future prices, so $excess_B$ increases.

Table 4 reports the estimates for the model in Equation 8. Column 1 ($excess_E$) shows how $excess_E$ is positively determined by its own lags. In column 2, $dprice_E$ is negatively associated with its substitute's lag and positively associated with its own lag, which is rather surprising. A plausible explanation might be that a large group of bids for E originates from Category B bidders. When $dprice_B$ is observed to increase in previous periods, individuals would expect the acceleration in B to fall in the next period and hence place their bids on B instead of E. This dampens $dprice_E$, thereby resulting in a negative relationship. As for the lags of $dprice_E$ being positively associated with $dprice_E$, the phenomenon is plausibly attributed to speculation, but it will be shown in the full VAR that this relationship is not very strong.

In column 3, $excess_B$ is negatively associated with lags of $dprice_B$ and positively associated with lags of $dprice_E$. With an acceleration in price, $excess_B$ would be smaller as bidders substitute to alternatives. $excess_B$ is positively related to its own lag, supporting an explanation similar to its counterpart in Table 3. In column 4, $dprice_B$ is negatively associated with its own lag and positively associated with the lag of its substitute. The negative coefficient on lag of $dprice_B$ is unsurprising as it suggests that the price increase in any period should result in a smaller price increase in the following period. Otherwise, there would be a continuous increase and acceleration in price. The positive coefficient on lag of $dprice_E$ is unsurprising as they are substitutes.

Looking at the matrix of contemporaneous relationships A, one unit increase in $excess_B$ increases $excess_E$ by 0.247. This implies substitution, as an increase in $excess_B$ is partially absorbed by its substitute through the corresponding increase in $excess_B$. Furthermore, $dprice_E$ is positively affected by $dprice_B$ as we expect the prices of various COE categories to move contemporaneously as they respond similarly to economic shocks. The positive effect of $dprice_B$ on $excess_B$ is followed from the speculation account.

5.2 SVAR of Categories A, B and E

This subsection will discuss the results of the full SVAR, i.e. vector of endogenous variables in Equation 9 applied to Equation 5. Estimates are in Tables 5 and 6, and the main intertemporal observations are summarized in Result 1, and contemporaneous observations are summarized in Result 2.

The joint significance of the variable's lags can be tested for Granger causality by the Wald test; the results are in Table 2.

Table 2 presents the Wald statistics for the joint test of lags for each variable. The dependent variable is indicated in the column name, and the explanatory variables are listed on the left for each model. In the analysis below, I first focus on lags with significant Wald statistics, then look at the corresponding result to determine the sign of the coefficient. Additionally, the regression output allows us to observe the sign and significance of a variable's own lags.

Consider Table 5: in column 1 ($excess_E$), we observe a positive association with lags of $excess_E$ and negative association with lags of $dprice_A$. This might be attributed to how higher dpriceA results in bidders exiting the market altogether, thereby reducing $excess_E$. In column 2 ($dprice_E$), Wald statistic suggests that the positive association with $dprice_A$ and the negative association with $dprice_B$ are significant. The account for $dprice_A$ is more intuitive: as $dprice_A$ is higher in previous periods, bidders switch to bidding for E and submit higher prices. The account for $dprice_B$ is identical to that given in Table 4. The results in column 3 ($excess_A$) are similar to our initial estimates in Table 3: $excess_A$ is positively correlated with its own lags and negatively correlated with $dprice_A$. In column 4 ($dprice_A$) we still observe a negative coefficient on $dprice_A$, and a positive coefficient on $dprice_E$, suggesting substitution from E to A intertemporally. In column 5 ($excess_B$), we observe the expected positive association with its own lag $excess_B$. There is a significant positive association with lags of $dprice_E$ and negative association with lags of $dprice_B$, which is intuitive, as people substitute into B when $price_B$ is lower and $price_E$ is higher. In column 6 ($dprice_B$), there is also an intuitive result: there are positive coefficients on lags of $dprice_A$ and $dprice_E$ and negative coefficients on lags of dpriceB.

Result 1. *Considering SVAR (3) of Category A, B and E COE, with brackets indicating a positive or negative relationship, dprice and excess share the following relationships:*

- *$excess_E$ is associated with lags of $dprice_A$ (-), $excess_E$ (+)*

- *$dprice_E$ is associated with lags of $dprice_A$ (+), $dprice_B$ (-)*
- *$excess_A$ is associated with lags of $dprice_A$ (-), $excess_A$ (+)*
- *$dprice_A$ is associated with lags of $dprice_A$ (-), $dprice_E$ (+)*
- *$excess_B$ is associated with lags of $dprice_B$ (-), $dprice_E$ (+), $excess_B$ (+)*
- *$dprice_B$ is associated with lags of $dprice_A$ (+), $dprice_B$ (-), $dprice_E$ (+)*

The LM test for serial correlation is conducted and it is found that there is no serial correlation on the first lag (test statistic of 42.0695 for $\chi^2(36)$), but there is some serial correlation in the second lag. The VAR also satisfies the stability condition as eigenvalues lie inside the unit circle.

The SVAR allows estimation of contemporaneous relationships captured in Table 6. The first row shows that $excess_E$ is positively affected by $excess_A$ and $excess_B$; the effect from B is stronger. As explained before, a shock that increases $excess_A$ or $excess_B$ can be absorbed by increasing $excess_E$ contemporaneously as bidders substitute into E. The second row shows that $dprice_E$ is positively affected by $dprice_A$ and $dprice_B$, which is unsurprising, as prices move together by arbitrage. Thus, even as bidders substitute from B to E due to a higher $price_B$, with the same willingness to pay for the COE, $dprice_E$ would also increase accordingly. The third row shows how $excess_A$ is positively affected by $dprice_A$ and $excess_B$. The former can be explained by risk aversion. The latter suggests that excessA may be more sensitive than $excess_B$ to shocks. The fourth column shows that $dprice_A$ is positively affected by both $excess_B$and $dprice_B$, as both can result in immediate substitution into bidding up $price_A$. Finally, $excess_B$ is positively associated with $dprice_B$, and the standard speculation account may be used.

Result 2. *Under Assumptions 2 and 3, SVAR (3) of Category A, B and E COE, with brackets indicating a positive or negative relationship, has the following contemporaneous relationships:*
- *$excess_E$ is associated with $excess_A$ (+), $excess_B$ (+)*
- *$dprice_E$ is associated with $dprice_A$ (+), $dprice_B$ (+)*

- *$excess_A$ is associated with $dprice_A$ (+), $excess_B$ (+)*
- *$dprice_A$ is associated with $excess_B$ (+), $dprice_B$ (+)*
- *$excess_E$ is associated with $dprice_B$ (+)*

Inspection of Result 2 evidence suggests that the variables move together contemporaneously, since the significant results are positively associated with each other. Errors are likely correlated across equations. Thus, if instruments were available, estimation by 3SLS is warranted.

5.3 *Robustness Checks*

For estimation with logarithms to be viable, one has to deal with the outliers. In this model, the top and bottom 1% of $\Delta \ln price$ observations in A, B and E are trimmed. The resulting Wald statistics are reported in Table 7, but the full VAR estimates are available upon request. Only the Wald statistic is reported as we are most interested in whether the results are still significant. Evidently, Result 1 is largely robust. Columns 1 to 5 show that the significant variables are exactly the same as that expected in Result 1. In column 6, we would expect $d\ln price_A$ to be significant in the Granger-causing variable $excess_E$. The Wald statistic for $d\ln price_A$ is high, which supports Result 1, though it is insignificant at 1% level (p-value is 0.011).

As mentioned in Section 2, there was a policy change in the demarcation of Category A and Category B cars. Consequently, there might be some structural break in September 2013. To observe if changing the category demarcation affects our results, a VAR is estimated for the period before September 2013, and another VAR is estimated for the period from September 2013 onwards. While it is arguable that the most accurate result is obtained by partitioning the series into the two time periods because the demarcations of categories are different, there is still merit in using the entire series from April 2002. Firstly, considering the number of parameters that we need to estimate in the VAR model, using a longer time series would ensure that we do not lose too many degrees of freedom. Secondly, if a model for forecasting is required, using a longer time series would benefit the reliability of the forecast. As such, partitioning into the two time periods is used more as a check for the robustness of our previous results.

The results for partitioning are reported in Table 8. Evidently, the Wald statistics are lower in Period 2 than Period 1, and there are less relationships that are significant at 1% level. This might be attributed to how Period 1 has a longer time series than Period 2, which results in the loss of power in Period 2 estimates. Everything that is significant in Period 2 is also significant in Period 1; thus, rather than the time series in both periods behave differently, it is more likely that there is simply low power in Period 2.

Table 9 reports the VAR results for partitioning. In column 1, while the Wald statistic is no longer significant for $dprice_E$, it is evident that the first lag of $dprice_E$ is still significant, which is consistent with Result 1. In column 2, significance is observed for lags of dpriceA, $dprice_B$, and $dprice_E$ in both periods, as expected. In column 3 we observe significance in Period 1 but not in Period 2: this weakens the reliability of our conclusions about the determinants of $dprice_E$ across both periods. In column 4 we still observe that lags of $dprice_A$ and $excess_A$ are significant in both periods, which is consistent with prior results. In column 5, while the Wald statistic is no longer significant for $dprice_E$, its first lag is still significant in Table 9, so the conclusion that $excess_B$ is related to lags of $dprice_B$, $dprice_E$ and $excess_B$ is still robust. In column 6, the Wald statistic for $dprice_A$ is significant in the first period, and is high but insignificant in the second period. However, when looking at Table 9, it is evident that lags of $dprice_A$ is still significant in both period. After looking at the partitioned results, it appears that the Result 1 findings are still evident in both periods, except for the determinants of $dprice_E$.

6 Conclusion

This paper develops an intertemporal micro-founded model to show how prices and number of excess bids of any COE category are simultaneously determined and depended on expectations that are formed by history. For a theoretical model of intertemporal and discrete choice of generally substitutable auctioned goods to be realistic in the COE analysis, the nature of the COE types must be scrutinized more carefully, and it is expected that Category A, B, and E COE's are not perfectly and symmetrically substitutable. By estimating an SVAR model, contemporaneous and intertemporal relationships between *dprice* and *excess* of various categories are found. The main results are summarized in Result 1 and 2 and are found to be robust to differing formulations.

The main contribution of this paper would be the development of a model for the COE game in Singapore and an empirical verification of auction theory. The theoretical propositions suggest that price and excess are simultaneously determined and are intertemporally affected by their substitutes. While there is insufficient evidence to support Proposition 1, empirical evidence supports Propositions 2 and 3 as variables in a COE category's history and its substitutes are significantly relevant. By looking at the relationship between COE categories and their consequent behavior, future policies in Singapore can also be formulated more effectively. Potential studies in the future could include a formal analysis of the auction system, refined by the nature of the good, and perhaps how such empirical evidence sheds light on Singaporeans' preferences for privately owned vehicles.

A further extension is to observe whether there might be long run convergence of these variables. While error correction models are typically estimated for nonstationary processes, Keele and De Boef (2004) points out that Vector Error Correction Models (VEC) can also be used for stationary variables, as it can be shown that any VAR can be written in VEC form, where for some p lags in VAR, there will be p-1 lags in VEC. VEC might be warranted on the grounds that, if a long-run relationship exists, we can separate the short-term and long-term effects in the model. Furthermore, considering how the change in price is negatively associated with its own lags, it seems like the time series might be converging to some long-run relationship. With the existing theory, we might expect, for instance, cointegration between priceB and priceE. However, such discussion is beyond the scope of this paper, as this paper does not aim to place restrictions on which variables are plausibly involved in a cointegrating relationship, so that we can allow all variables to interact freely in the model. Hence, it remains a possibility for future research.

7 References

Adhikari, R. and R. K. Agrawal (2013). An Introductory Study on Time Series Modeling and Forecasting. *arXiv preprint arXiv:1302.6613*.

Chu, S. (2012). Allocation Flexibility and Price Efficiency Within Singapore's Vehicle Quota System. *Transportation Research Part A: Policy and Practice 46*(10), 1541–1550.

Chu, S. (2018). Singapore's Vehicle Quota System and its Impact on Motorcycles. *Transportation 45*(5), 1419–1432.

Hana´k, T. and P. Muchov´a (2015). Impact of Competition on Prices in Public Sector Procurement. *Procedia Computer Science 64*, 729–735.

Huang, N., J. Li, and A. Ross (2018). The Impact of the Cost of Car Ownership on the House Price Gradient in Singapore. *Regional Science and Urban Economics 68*, 160–171.

Hungria-Gunnelin, R. et al. (2013). Impact of Number of Bidders on Sale Price of Auctioned Condominium Apartments in Stockholm. *International Real Estate Review 16*(3), 274–295.

Jofre-Bonet, M. and M. Pesendorfer (2000). Bidding Behavior in a Repeated Procurement Auction: A Summary. *European Economic Review 44*(4-6), 1006–1020.

Keele, L. and S. De Boef (2004). Not Just for Cointegration: Error Correction Models with Stationary Data. *Documento de Trabajo. Departamento de Pol´ıtica y Relaciones Internacionales, Nuffield College y Oxford University.*

Kilian, L. (2011). Structural Vector Autoregressions.

Koh, W. and D. Lee (1993). Auctions for Transferable Objects: Theory and Evidence from the Vehicle Quota System in Singapore. *Asia Pacific Journal of Management 10*(2), 177–194.

Koh, W. T. (2003). Control of Vehicle Ownership and Market Competition: Theory and Singapore's Experience with the Vehicle Quota System. *Transportation Research Part A: Policy and Practice 37*(9), 749–770.

Koh, W. T., R. S. Mariano, and Y. K. Tse (2007). Open vs. Sealed-bid auctions: Testing for Revenue Equivalence Under Singapore's Vehicle Quota System. *Applied Economics 39*(1), 125–134.

Mierendorff, K. (2013). The Dynamic Vickrey Auction. *Games and Economic Behavior 82*, 192–204.

Phillips, P. C. and P. Perron (1988). Testing For a Unit Root in Time Series Regression. *Biometrika 75*(2), 335–346.

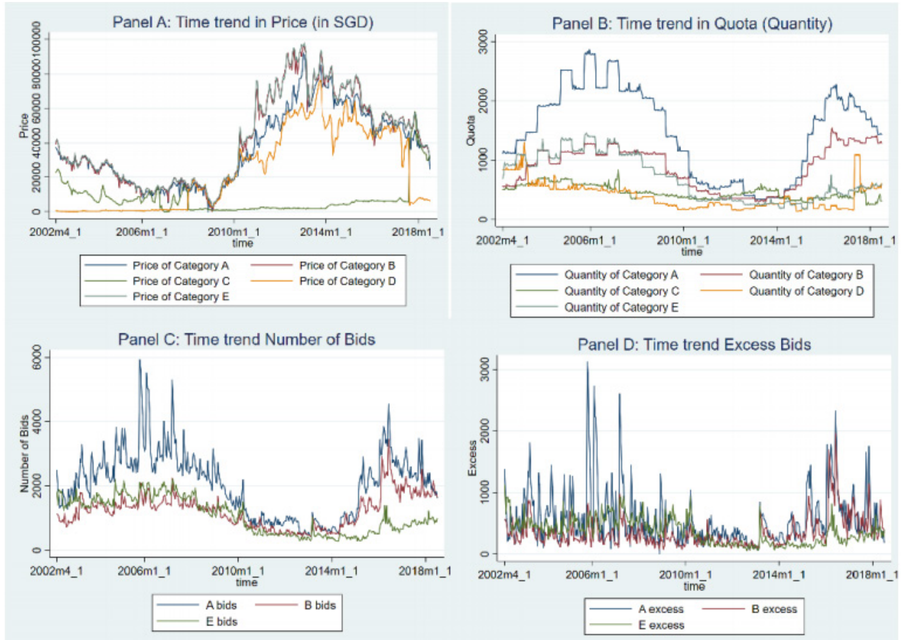
Samuelson, P. A. (1937). A Note on Measurement of Utility. *The review of economic studies 4*(2), 155–161.

Sims, C. A. (1980). Macroeconomics and Reality. *Econometrica: Journal of the Econometric Society*, 1–48.

SGCharts. (2018). COE Prices Since 2002. Retrieved July 10, 2018 from https://coe.sgcharts.com/ Figure 1: Time trends

8 Appendix

Figure 1: Time trends



Note: The time labels are denoted as year, then month, then bidding session. For instance, the first bid of January 2018 will be denoted 2018m1 1.

Table 1: Stationary Tests with 5 Lags

VARIABLES	(1) ADF	(2) PP. Z(rho)	(3) PP. Z(t)
priceA	-1.011	-2.916	-1.176
priceB	-1.058	-3.341	-1.276
priceE	-1.183	-3.511	-1.306
excessA	-5.223***	-156.527***	-9.928***
excessB	-3.958***	-103.604***	-7.799***
excessE	-3.568***	-74.014***	-6.853***

*** denotes that p-value is less than 0.01. t statistics are displayed.

Table 2: Granger Causality

		(1) dpriceA	(2) dpriceB	(3) dpriceE	(4) excessA	(5) excessB	(6) excessE
Model	dpriceA	-	63.348*	43.05*	58.575*	10.175	18.494*
	dpriceB	2.7977	-	16.901*	1.4109	37.384*	2.2717
	dpriceE	23.005*	64.364*	-	7.5058	26.408*	0.45015
	excessA	0.53784	8.1327	5.7598	-	0.79495	10.784
	excessB	6.2003	10.442	10.143	4.5443	-	8.7297
	excessE	3.5485	7.8209	10.999	0.59177	2.1568	-

*1% significance level for Granger causality Wald test

Table 3: SVAR for B's price and excess

VARIABLES	(1) excessB	(2) dpriceB
L.excessB	0.690***	2.965**
	(0.0529)	(1.229)
L2.excessB	0.100*	-2.867**
	(0.0530)	(1.230)
L.dpriceB	-0.0109***	-0.0872
	(0.00229)	(0.0532)
L2.dpriceB	-0.00711***	-0.0171
	(0.00231)	(0.0536)
Constant	72.29***	-58.36
	(15.44)	(358.5)
Observations	388	388

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

VARIABLES	(1) excessB	(2) dpriceB
excessB	1	-0.0143***
	(0)	(0.00206)
dpriceB	0	1
	(0)	(0)

Note: The first table shows the underlying VAR estimates. The second table shows the estimates of matrix **A***, where* $\mathbf{Ay}_t$ *is on the LHS of structural form VAR in Equation 5.*

Table 4: SVAR between B and E

VARIABLES	(1) excessE	(2) dpriceE	(3) excessB	(4) dpriceB
L.excessE	0.469***	3.711***	-0.00188	3.391**
	(0.0530)	(1.356)	(0.0719)	(1.581)
L2.excessE	0.291***	-2.349	0.0654	-1.823
	(0.0575)	(1.471)	(0.0780)	(1.715)
L3.excessE	0.109**	-0.854	-0.0985	-1.168
	(0.0527)	(1.350)	(0.0716)	(1.574)
L.dpriceE	-0.00454	0.478***	0.0278***	1.057***
	(0.00450)	(0.115)	(0.00610)	(0.134)
L2.dpriceE	-0.00347	-0.183	0.00834	0.248*
	(0.00474)	(0.121)	(0.00643)	(0.141)
L3.dpriceE	-0.000579	-0.191*	0.00501	-0.128
	(0.00418)	(0.107)	(0.00567)	(0.125)
L.excessB	0.0458	1.941*	0.684***	2.579**
	(0.0404)	(1.035)	(0.0549)	(1.207)
L2.excessB	-0.0674	-3.308***	0.00357	-3.350**
	(0.0482)	(1.234)	(0.0654)	(1.438)
L3.excessB	-0.00293	1.568	0.140**	1.174
	(0.0403)	(1.032)	(0.0547)	(1.203)
L.dpriceB	-0.00116	-0.210**	-0.0322***	-0.891***
	(0.00391)	(0.100)	(0.00531)	(0.117)
L2.dpriceB	0.00301	0.143	-0.0173***	-0.345***
	(0.00435)	(0.111)	(0.00591)	(0.130)
L3.dpriceB	0.00135	0.0874	-0.0114**	-0.0517
	(0.00380)	(0.0973)	(0.00516)	(0.113)
Constant	54.69***	-266.5	72.77***	-300.4
	(15.14)	(387.6)	(20.55)	(451.7)
Observations	387	387	387	387

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

VARIABLES	(1) excessE	(2) dpriceE	(3) excessB	(4) dpriceB
excessE	1	-0.000268	-0.247***	0.00246
	(0)	(0.00434)	(0.0368)	(0.00377)
dpriceE	0	1	0.443	-0.779***
	(0)	(0)	(0.431)	(0.0196)
excessB	0	0	1	-0.0123***
	(0)	(0)	(0)	(0.00223)
dpriceB	0	0	0	1
	(0)	(0)	(0)	(0)

Table 5: VAR for ABE

VARIABLES	(1) excessE	(2) dpriceE	(3) excessA	(4) dpriceA	(5) excessB	(6) dpriceB
L.excessE	0.445***	4.277***	0.0648	1.750	0.0349	4.172***
	(0.0537)	(1.332)	(0.159)	(1.352)	(0.0739)	(1.516)
L2.excessE	0.279***	-3.001**	0.0150	-2.611*	0.0274	-2.479
	(0.0570)	(1.414)	(0.169)	(1.436)	(0.0785)	(1.610)
L3.excessE	0.122**	-0.389	-0.00817	0.737	-0.0976	-0.716
	(0.0526)	(1.305)	(0.156)	(1.325)	(0.0724)	(1.485)
L.dpriceE	-0.00215	0.385***	0.0300**	0.461***	0.0311***	0.924***
	(0.00444)	(0.110)	(0.0131)	(0.112)	(0.00610)	(0.125)
L2.dpriceE	-0.00261	-0.246**	0.00761	-0.114	0.00765	0.170
	(0.00464)	(0.115)	(0.0137)	(0.117)	(0.00638)	(0.131)
L3.dpriceE	-0.000401	-0.180*	-0.0101	-0.0376	0.00448	-0.119
	(0.00409)	(0.102)	(0.0121)	(0.103)	(0.00563)	(0.116)
L.excessA	0.0375*	-1.339**	0.645***	-0.103	0.00192	-1.803***
	(0.0227)	(0.563)	(0.0672)	(0.572)	(0.0312)	(0.641)
L2.excessA	0.0395	0.978	-0.0763	0.471	0.0255	0.939
	(0.0270)	(0.669)	(0.0799)	(0.679)	(0.0371)	(0.762)
L3.excessA	-0.0309	-0.349	0.00270	-0.241	-0.0203	-0.232
	(0.0227)	(0.563)	(0.0672)	(0.571)	(0.0312)	(0.640)
L.dpriceA	-0.0101***	0.379***	-0.0585***	-0.239***	-0.00861**	0.529***
	(0.00259)	(0.0643)	(0.00768)	(0.0653)	(0.00357)	(0.0732)
L2.dpriceA	-0.00843***	0.320***	-0.0195**	-0.0898	0.00582	0.429***
	(0.00312)	(0.0775)	(0.00925)	(0.0786)	(0.00430)	(0.0882)
L3.dpriceA	-0.00223	0.186***	-0.00482	-0.0610	0.00264	0.255***
	(0.00286)	(0.0709)	(0.00846)	(0.0719)	(0.00393)	(0.0807)
L.excessB	0.0193	3.014***	0.221	-0.408	0.728***	3.907***
	(0.0464)	(1.152)	(0.138)	(1.169)	(0.0639)	(1.311)
L2.excessB	-0.129**	-3.689***	-0.0624	-2.570*	-0.0861	-3.265**
	(0.0563)	(1.397)	(0.167)	(1.418)	(0.0775)	(1.590)
L3.excessB	0.0279	1.927*	0.0506	2.386**	0.182***	1.323
	(0.0465)	(1.154)	(0.138)	(1.172)	(0.0640)	(1.314)
L.dpriceB	0.00275	-0.341***	-0.0124	-0.107	-0.0331***	-1.071***
	(0.00394)	(0.0978)	(0.0117)	(0.0993)	(0.00543)	(0.111)
L2.dpriceB	0.00676	-0.00389	-0.00251	0.0531	-0.0183***	-0.550***
	(0.00451)	(0.112)	(0.0134)	(0.114)	(0.00621)	(0.127)
L3.dpriceB	0.00252	-0.00247	0.000383	-0.0256	-0.0131**	-0.169
	(0.00381)	(0.0946)	(0.0113)	(0.0960)	(0.00525)	(0.108)
Constant	56.65***	-374.8	144.4***	162.8	69.75***	-441.6
	(14.78)	(366.7)	(43.77)	(372.3)	(20.34)	(417.4)
Observations	387	387	387	387	387	387

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 6: Contemporaneous Relationships for ABE

VARIABLES	(1) excessE	(2) dpriceE	(3) excessA	(4) dpriceA	(5) excessB	(6) dpriceB
excessE	1	0.00130	-0.0802***	-0.00119	-0.125***	0.000121
	(0)	(0.00420)	(0.0209)	(0.00241)	(0.0437)	(0.00377)
dpriceE	0	1	0.209	-0.0923***	0.609	-0.750***
	(0)	(0)	(0.253)	(0.0287)	(0.528)	(0.0250)
excessA	0	0	1	-0.0251***	-1.131***	-0.00477
	(0)	(0)	(0)	(0.00563)	(0.0891)	(0.00503)
dpriceA	0	0	0	1	-2.695***	-0.469***
	(0)	(0)	(0)	(0)	(0.792)	(0.0386)
excessB	0	0	0	0	1	-0.0153***
	(0)	(0)	(0)	(0)	(0)	(0.00235)
dpriceB	0	0	0	0	0	1
	(0)	(0)	(0)	(0)	(0)	(0)

Table 7: VAR logarithmic specification

	(1) dlnpriceA	(2) dlnpriceB	(3) dlnpriceE	(4) excessA	(5) excessB	(6) excessE
dlnpriceA	-	15.574*	12.364*	68.706*	2.5036	11.043
dlnpriceB	4.1659	-	4.6013	3.8979	30.097*	10.234
dlnpriceE	29.63*	50.557*	-	4.0885	12.909*	3.8907
excessA	3.7799	7.8603	10.414	-	4.1967	12.689*
excessB	3.6183	3.8832	8.5045	7.6516	-	10.761
excessE	1.4262	2.7845	8.1243	1.6348	2.0381	-

*1% significance level for Granger causality Wald test

Table 8: VAR Partitioned into Two Periods

		(1) dpriceA	(2) dpriceB	(3) dpriceE	(4) excessA	(5) excessB	(6) excessE
Period 1	dpriceA	-	43.502*	29.2*	35.423*	12.626*	14.762*
	dpriceB	4.49	-	23.276*	0.68465	13.993*	1.3148
	dpriceE	17.725*	49.804*	-	2.625	14.313*	0.07828
	excessA	0.17145	7.8355	5.9849	-	2.581	7.2549
	excessB	1.9295	8.2368	5.014	0.27138	-	5.3429
	excessE	5.1349	3.0502	6.2601	0.38423	1.0689	-
Period 2	dpriceA	-	13.6*	7.1099	31.677*	0.12561	5.9025
	dpriceB	7.985	-	4.5263	0.0615	10.994*	0.1657
	dpriceE	6.8242	10.026*	-	5.3399	6.778	1.806
	excessA	2.3898	1.6675	0.13613	-	9.0864	7.0362
	excessB	1.7028	0.81289	0.17997	0.27617	-	0.57347
	excessE	0.28503	0.08004	0.54791	2.6186	5.0457	-

Note: In both sets of regressions. two lags are used. The choice of two lags is given by the minimum AIC and HQIC in both partitions. Period 1 refers to the series before September 2013 and Period 2 refers to September 2013 onwards.

Table 9: VAR Partitioned into Two Periods

VARIABLES	(1) dpriceA	(2) dpriceB	(3) dpriceE	(4) excessA	(5) excessB	(6) excessE
L.dpriceA	-0.216***	0.575***	0.403***	-0.0562***	-0.0117***	-0.0120***
	(0.0733)	(0.0881)	(0.0770)	(0.00946)	(0.00331)	(0.00332)
L.dpriceB	-0.100	-1.108***	-0.428***	-0.0108	-0.0174***	0.00241
	(0.103)	(0.124)	(0.109)	(0.0134)	(0.00467)	(0.00468)
L.dpriceE	0.427***	0.985***	0.475***	0.0241	0.0200***	-0.00134
	(0.117)	(0.141)	(0.123)	(0.0151)	(0.00529)	(0.00530)
L.excessA	0.214	-2.120***	-1.636**	0.696***	0.0440	0.0425
	(0.637)	(0.766)	(0.669)	(0.0822)	(0.0287)	(0.0288)
L.excessB	-2.542	6.485***	4.451**	-0.00237	0.473***	-0.0947
	(1.898)	(2.283)	(1.994)	(0.245)	(0.0857)	(0.0859)
L.excessE	1.974	2.405	3.376**	0.113	0.0547	0.557***
	(1.419)	(1.707)	(1.491)	(0.183)	(0.0640)	(0.0642)
Constant	1,104**	-232.8	-7.660	191.2***	109.5***	95.50***
	(455.3)	(547.8)	(478.4)	(58.80)	(20.55)	(20.61)
Observations	271	271	271	271	271	271

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

VARIABLES	(1) dpriceA	(2) dpriceB	(3) dpriceE	(4) excessA	(5) excessB	(6) excessE
L.dpriceA	-0.395***	0.282**	0.149	-0.0710***	-0.00316	-0.00799**
	(0.135)	(0.137)	(0.121)	(0.0130)	(0.00919)	(0.00353)
L.dpriceB	0.0118	-0.469**	0.0923	0.00214	-0.0425***	0.00117
	(0.195)	(0.197)	(0.174)	(0.0187)	(0.0132)	(0.00508)
L.dpriceE	0.560***	0.676***	0.309	0.0402*	0.0325**	-0.00653
	(0.214)	(0.217)	(0.192)	(0.0206)	(0.0146)	(0.00560)
L.excessA	1.998	-1.771	-0.0123	0.744***	-0.0458	0.0938***
	(1.358)	(1.372)	(1.215)	(0.130)	(0.0923)	(0.0355)
L.excessB	-2.491	1.336	-0.559	0.0989	0.845***	0.0348
	(1.973)	(1.993)	(1.765)	(0.190)	(0.134)	(0.0515)
L.excessE	1.614	0.863	2.005	-0.275	0.0416	0.276***
	(3.033)	(3.064)	(2.714)	(0.291)	(0.206)	(0.0792)
Constant	-1,319**	-582.7	-786.7	37.13	-5.275	14.42
	(639.4)	(645.8)	(572.0)	(61.42)	(43.47)	(16.70)
Observations	117	117	117	117	117	117

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: In both sets of regressions. two lags are used, but only the first lag is required. The choice of two lags is given by the minimum AIC and HQIC in both partitions. The top panel shows results for the series before September 2013 and the bottom panel is for September 2013 onwards.

[THIS PAGE INTENTIONALLY LEFT BLANK]

Contests with Uncertainty on Success Functions: Theory and Applications

Alvaro Enrique Aguirre Moya
Universidad Carlos III de Madrid
University of California, Berkeley
London School of Economics

Abstract

In this paper, we explore the possibility of analyzing contests where agents are uncertain about the Contest Success Functions they are facing. We present a model with the inclusion of a *fairness parameter* that captures the probability of the contest to be determined by a Tullock ratio versus a simple lottery, through a convex combination of both. After a brief analysis of complete information, we focus on an incomplete information setting that leads to the characterization of a Bayesian equilibrium with particular conditions for its existence. Furthermore, a detailed comparative static analysis is carried out. Finally, we present an application of the model to electoral competition with heterogeneity in voters and an information advantage.

Acknowledgements

I am very grateful to my advisor, Prof. Luis Corchón, for his guidance through my first theoretical paper. I would also like to thank Prof. Carmen Beviá for her useful comments.

Table of Contents

1 Introduction

In a contest, a set of agents exerts costly efforts in order to increase their probabilities of winning a prize. There is no shortage of these situations in real life. Consider advertising by rival firms, patent races, sports competitions, electoral campaigns, lobbying, and beauty pageants, amongst others. The pioneering work in this area can be attributed to Gordon Tullock (1967), who first studied monopolies and how the rent obtained by them actually dissipates through the battle for monopoly rights. Unlike standard microeconomics, which assumes there are well-defined property rights, endowments, and a given technology, contest theory attempts to explain how these were obtained in the first place.

Contest Success Functions (CSF), a key component in contests, translate the efforts exerted by each agent into winning probabilities. The Tullock (1980) ratio form has been the most commonly used in the literature because of its analytical properties, axiomatized by Skaperdas (1996), and the possibility of finding a Nash equilibrium. The contrasting work of Hirshleifer (1989) with *difference CSF* has also provided valuable insight in the field, although no Nash equilibrium can be found. While a wide variety of contests have been explored since the popularization of the field, most papers considered the CSF as public information known by all contenders.

Several authors have analyzed success functions, attempting to provide a perspective more applicable to real life. Jia (2008) proposed stochastic foundations for the ratio CSF, basing his analysis on underlying random shocks. This idea was further developed in Jia et al. (2013), where four methods of deriving a CSF were analyzed: stochastic, axiomatic, optimal-design, and positive-microfoundations. Other authors have introduced insights from behavioral economics into contest theory, like Bahard & Nitzan (2008) with Cumulative Prospect Theory, and Anderson, Goeree & Holt (1998) with a bounded rationality setup.

We consider that in real life, success functions tend to be unknown. There is no perfect information on how sensitive winning probabilities are with respect to efforts, and the best approximation to analyze them relies on probability distributions. A general treatment of incomplete information in Tullock-ratio contests was developed in Einy et al. (2015), where the existence of a Bayesian Nash equilibrium was proved. This paper is particularly relevant to our discussion, since it includes success functions into the measurable space of states of nature. Until then, Bayesian equilibriums in contests had only considered information asymmetries in valuation of the prize (Malueg & Yates (2004), Ryvkin (2010), Gallice (2014)) and in the cost of efforts (Fey (2008), Singh & Wittman (1988)). Unlike most of the work previously done in the field, we propose an analytical resolution to a setting where agents are uncertain about the success functions they are facing. Furthermore, we include uncertainty regarding the probability distribution. We focus on one-sided asymmetric information contests, where one agent has an advantage of knowing the true probability distribution of the CSF, while the other relies on expected values. This setting follows the discussion in Hurley & Shogren (1998), where it is argued that one-sided information asymmetries are sufficient to capture contest behavior.

The paper proceeds as follows: Section 2 introduces the model in a complete and incomplete information setting, followed by an exhaustive analysis of the comparative statics. Section 3 proposes an application of our model to an electoral competition with heterogeneity in voters. Finally, Section 4 summarizes our findings and presents potential extensions to our model. An appendix is included with the algebra and proofs used through the paper.

2 The Model

We start by introducing a model with N=2 risk-neutral contestants for simplicity. The i-th contestant individually exerts an effort $e_i \in R_+$ in order to increase his probabilities, $p_i(e_i,e_{-i})$, of winning a prize valued at V_i, where $i \in \{1,2\}$. Note that the subscript $-i$ denotes the effort exerted by the opponent in the contest. If the prize is indivisible, like in patent races, sport tournaments, or litigation, p_i represents the probability of winning, whereas in contests with divisible prizes, like war over territory, it represents the share of the prize won by the i^{th} contestant. The individual efforts are mapped to $p_i(e_i,e_{-i})$ following the generalization of the Contest Success Functions (CSF) for some unknown value r that follows a given probabilistic distribution, $F(r)$:

$$p_i(e_i, e_{-i}) = \begin{cases} \frac{e_i^r}{\sum_{j=1}^N e_j^r} & if \quad \sum_{j=1}^N e_j^r \neq 0 \\ \frac{1}{N} & if \quad \sum_{j=1}^N e_j^r = 0 \end{cases} \quad with\ r \sim F(r),\ i, \in \{1,2\}$$

Each contestant's probability of winning is determined by how large his effort is, relative to the aggregate effort of all agents, assuming at least one exerts a positive effort. Otherwise, the probability of winning is equal among all competitors. It is then straightforward to see that $p_i(e_i,e_{-i})$ is non-decreasing in e_i and non-increasing in e_{-i}. One of the key properties of this type of function, analyzed in Corchón (2007) and Skaperdas (1996), is that it is homogeneous of degree zero, such that for a vector of efforts E: $p_i(E) = p_i(\lambda E)$, $\forall \lambda > 0$, $\forall i \in N$. This means that the unit in which effort is measured is irrelevant to the winning probabilities, which is a desirable characteristic for a CSF.

The exponent r, a central piece of our analysis, represents how sensitive the probability of winning is to the contestants' efforts. When r=0, we have a case where, no matter how much agents invest in the contest, the CSF will always be a symmetric lottery with $p_i(e_i,e_{-i}) = 1/N$. The case when $r = 1$, commonly used in contest theory literature, is the Tullock ratio (1980), where each agent's probability of winning is determined by the ratio of their effort to the total effort exerted in the contest.

As $r \to \infty$, the CSF loses its noise component and the slightest difference in efforts yields a great difference in winning probabilities. Amegashie (2006) proposed a more tractable CSF with a noise parameter α and exponent r that attempt to explain the response of p_i to changes in (e_i,e_{-i}). A generalization of this model where every agent has an independent noise parameter can be seen in the work of Corchón and Dahm (2010). Similar intuition can be found in the work of Franke et al. (2014), where the role of an exogenous "head start", acting as idiosyncratic noise, affects the contest equilibrium.

We attempt to include uncertainty on how the agents' efforts are translated into winning probabilities by taking r as a stochastic component with a given probability distribution $F(r)$. For simplicity and tractability, in this paper, we are going to assume that the probabilistic distribution is discrete and that r can only take the values of 0 (lottery) or 1 (Tullock ratio). We will use the term fair contest to describe the scenario where r=1, and unfair contest for the case when r = 0.

We propose a parameter $\psi \in [0,1]$ that captures the probability that the contest will be fair. Therefore, $p_i(e_i,e_{-i})$ will be a convex combination between a Tullock ratio and a lottery, respectively weighted by $\{\psi, 1-\psi\}$. The expected payoffs for each agent will then be their expected probability of winning, times their valuation of the prize, minus the cost of effort, which we will assume, without loss of generality, to be marginally constant and equal to one. Hence:

$$E[\Pi_i]=[(1-\psi)\ 1/2+\psi\ e_i/(e_i+e_{-i})]\ V_i-e_i \quad \text{for } i \in \{1,2\}$$

A similar distribution rule was proposed in Nitzan (1991), where he argued that a share of the prize could be distributed on egalitarian grounds, while the rest would depend on relative efforts. Our model is conceptually different, since we make no assumptions about the prize's characteristics. We will address this problem from two perspectives. First, we analyze a *complete information* setting where both agents know the probability distribution, F(r), in other words, both know ψ. Then, we analyze an *incomplete information* setting where two possible values of ψ are presented and only one agent knows which one is the realized.

2.1 *Equilibrium with Complete Information on Probability Distributions*

We will first briefly approach the situation from a complete information perspective, where both contestants know the true value of the parameter ψ. Taking the first order conditions from the expected payoffs functions yields:

$$\frac{\partial \mathrm{E}[\Pi_i]}{\partial e_i} = 0 \leftrightarrow e_i = \frac{(e_i + e_{-i})^2}{\psi V_{-i}} \text{ for } i \in \{1,2\}$$

Solving the system of equations obtained by the i first order conditions:

We have a profit-maximizing effort, since second order condi-

$$e_i^* = \frac{\psi V_i^2 V_{-i}}{\left(\sum_{k=1}^{N} V_k\right)^2} \text{ for } i \in \{1,2\}$$

tions holds: $(\partial^2 E[\Pi_i])/(\partial e_i^2) < 0$. The probabilities of winning for each contestant are given by:

$$p_i(e_i^*, e_{-i}^*) = (1-\psi)\frac{1}{2} + \psi\left(\frac{V_i}{\sum_{k=1}^{N} V_k}\right) \text{ for } i \in \{1,2\}$$

Comparing these results with traditional contest theory literature, as described in Corchón's (2007) survey, it is easy to see that the ψ parameter acts as a scalar of the efforts of every contestant. Moreover, the winning probabilities for any contestant here are the convex combination of a symmetric lottery and the contestant's relative valuation of the prize.

Proposition 1. *If both agents know the true value of ψ, the optimal effort exerted by each candidate is the effort under a certain Tullock ratio, $(V_i^2 V_{-i})/[(\sum_{k=1}^{N} V_k)^2]$, scaled by the parameter ψ.*

2.2 *Bayesian Equilibrium*

In our setting of incomplete information, the fairness parameter ψ can take two values $\{\underline{\psi}, \overline{\psi}\}$ with probabilities $\{1\text{-}p, p\}$ respectively, where $\underline{\psi} < \overline{\psi}$. We assume that contestant 2 has full information about the probability distribution of r and, therefore, knows the true realization of ψ, while contestant 1 only relies on the probabilities of $\{\underline{\psi}, \overline{\psi}\}$. In order to characterize the Bayesian equilibrium, contestant 1 will maximize his expected profits based on the efforts exerted by player 2 in each of the possible states of nature, which we will respectively call $\{\underline{e}^2, \overline{e_2}\}$. We will proceed to find the three first order conditions that yield the equilibrium. For the agent with full information:

If $\psi = \underline{\psi}$:

$$\mathrm{E}[\underline{\Pi_2}] = \left[(1-\underline{\psi})\tfrac{1}{2} + \underline{\psi}\frac{e_2}{e_1 + \underline{e_2}}\right] V_2 - \underline{e_2}$$

The first order conditions yields:

$$\frac{\partial \mathrm{E}[\underline{\Pi_2}]}{\partial \underline{e_2}} = 0 \leftrightarrow \underline{\psi}\frac{e_1}{(e_1 + \underline{e_2})^2} V_2 = 1 \leftrightarrow \underline{e_2} = \sqrt{\underline{\psi} V_2 e_1} - e_1 \;\text{(i)}$$

Likewise, if $\psi = \overline{\psi}$:

$$\mathrm{E}[\overline{\Pi}_2] = \left[(1-\overline{\psi})\tfrac{1}{2} + \overline{\psi}\frac{e_2}{e_1 + \overline{e_2}}\right] V_2 - \overline{e_2}$$

$$\frac{\partial \mathrm{E}[\overline{\Pi}_2]}{\partial \overline{e_2}} = 0 \leftrightarrow \overline{\psi}\frac{e_1}{(e_1 + \overline{e_2})^2} V_2 = 1 \leftrightarrow \overline{e_2} = \sqrt{\overline{\psi} V_2 e_1} - e_1 \;\text{(ii)}$$

It is easy to see that second order conditions hold in both cases so we have a profit maximizing result, since $(\partial^2 E[\Pi_2])/(\partial e_2^2) < 0$ for the high and low ψ. Agent 1 will maximize his expected profits depending on the probabilities that each state will occur. Again, second order conditions hold, ensuring a prof-

$$\mathrm{E}[\Pi_1] = p\left[\left((1-\overline{\psi})\frac{1}{2} + \overline{\psi}\frac{e_1}{e_1 + \overline{e_2}}\right)V_1 - e_1\right] + (1-p)\left[\left((1-\underline{\psi})\frac{1}{2} + \underline{\psi}\frac{e_1}{e_1 + \underline{e_2}}\right)V_1 - e_1\right]$$

it-maximizing effort.

$$\frac{\partial \mathrm{E}[\Pi_1]}{\partial e_1} = 0 \leftrightarrow p\overline{\psi}\frac{\overline{e_2}}{(e_1 + \overline{e_2})^2} V_1 + (1-p)\underline{\psi}\frac{\underline{e_2}}{(e_1 + \underline{e_2})^2} V_1 = 1 \;\text{(iii)}$$

Finding the first order condition:

Solving (i), (ii) and (iii) simultaneously gives us the Bayesian Equilibrium. The extensive algebra can be found in the appendix of the paper. The optimal efforts are:

$$e_1^* = \frac{\left(p\sqrt{\overline{\psi}}+(1-p)\sqrt{\underline{\psi}}\right)^2 V_1^2 V_2}{(V_1+V_2)^2}$$

$$\overline{e_2}^* = \frac{\left(p\sqrt{\overline{\psi}}+(1-p)\sqrt{\underline{\psi}}\right)V_1V_2}{(V_1+V_2)}\left[\sqrt{\overline{\psi}} - \frac{\left(p\sqrt{\overline{\psi}}+(1-p)\sqrt{\underline{\psi}}\right)V_1}{(V_1+V_2)}\right] \qquad \underline{e_2^*} = \frac{\left(p\sqrt{\overline{\psi}}+(1-p)\sqrt{\underline{\psi}}\right)V_1V_2}{(V_1+V_2)}\left[\sqrt{\underline{\psi}} - \frac{\left(p\sqrt{\overline{\psi}}+(1-p)\sqrt{\underline{\psi}}\right)V_1}{(V_1+V_2)}\right]$$

These algebraic expressions, that initially might seem complicated, make more sense when analyzed carefully. For notation, we will call the weighted product α ⍰ $(p\sqrt{\overline{\psi}}+(1\text{-}p)\sqrt{\underline{\psi}})$, which is

$$e_1^* = \frac{\alpha^2 V_1^2 V_2}{(V_1+V_2)^2} \quad , \quad \overline{e_2}^* = \frac{\alpha V_1 V_2}{(V_1+V_2)}\left[\sqrt{\overline{\psi}} - \frac{\alpha V_1}{(V_1+V_2)}\right] \quad , \quad \underline{e_2^*} = \frac{\alpha V_1 V_2}{(V_1+V_2)}\left[\sqrt{\underline{\psi}} - \frac{\alpha V_1}{(V_1+V_2)}\right]$$

key for our analysis. This way:
Since by definition e_i ⍰ R_+ ⍰ i ⍰ {1,...,N}, we need an additional condition for the existence of an inner solution in our Bayesian equilibrium. Given the nature of the parameters, e_1^* always

$$\overline{e_2}^* \geq 0 \leftrightarrow \frac{\sqrt{\overline{\psi}}}{\alpha} \geq \frac{V_1}{(V_1+V_2)} \quad \text{and} \quad \underline{e_2^*} \geq 0 \leftrightarrow \frac{\sqrt{\underline{\psi}}}{\alpha} \geq \frac{V_1}{(V_1+V_2)}$$

holds as a positive quantity. For the efforts of agent 2:
It is sufficient to assume the latter, since it is the most restrictive condition. A proof of this can be found in the appendix.

Proposition 2. The existence of an inner solution in the Bayesian equilibrium is conditional on:

$$\sqrt{\underline{\psi}}/\alpha \geq V_1/(V_1+V_2) \tag{2.1}$$

Given the construction of the left-hand-side term, it is bounded between 0 and 1, which implies that, if the valuation of the contestant with incomplete information is at least twice as big as the valuation of the contestant with complete information, an inner equilibrium is unfeasible. Moreover, the term will approach 0 as $(\overline{\psi} - \underline{\psi})$ ⍰ 1, since: $\lim_{\{\overline{\psi}-\underline{\psi}⍰1\}} \sqrt{\underline{\psi}}/\alpha = 0$. This means that, when the two states of nature are very different from each other, V_2 has to be very large in order for an inner equilibrium to hold.

Corollary. *If we have a case of symmetric valuations $V_1 = V_2$, the condition (2.1) turns into: $1 + 1/p \geq \sqrt{\overline{\psi}}/\sqrt{\underline{\psi}}$, so the smaller the probability of the fair scenario, p, the more likely it is for an inner Bayesian Equilibrium to hold.*

The winning probabilities of both agents will depend on the

$$p_1(e_1^*, \overline{e_2}^*) = (1-\overline{\psi})\frac{1}{2} + \frac{\alpha V_1}{\sqrt{\overline{\psi}}(V_1+V_2)}, \; p_2(e_1^*, \overline{e_2}^*) = (1-\overline{\psi})\frac{1}{2} + \overline{\psi}\left[1 - \frac{\alpha V_1}{\sqrt{\overline{\psi}}(V_1+V_2)}\right]$$

$$p_1\left(e_1^*, \underline{e_2^*}\right) = \left(1-\underline{\psi}\right)\frac{1}{2} + \frac{\alpha V_1}{\sqrt{\underline{\psi}}(V_1+V_2)}, \; p_2\left(e_1^*, \underline{e_2^*}\right) = \left(1-\underline{\psi}\right)\frac{1}{2} + \underline{\psi}\left[1 - \frac{\alpha V_1}{\sqrt{\underline{\psi}}(V_1+V_2)}\right]$$

realized state of nature. We find that:

Proposition 3. *When p=1 or p=0, and when $\overline{\psi} = \underline{\psi}$, the Bayesian Equilibrium found here is equal to the Nash Equilibrium found in subsection 2.1*

2.3 Comparative Statics and Analysis

2.3.1 Agent with Incomplete Information

We will first take a look at the agent with incomplete information, contestant 1. The comparative statics for this agent are more intuitive and simpler than what we will see for the contestant with complete information. The extensive algebra for all the statements presented in this section can be found in the appendix of the paper.

In traditional contest theory literature, as seen in Corchón (2007), agents with a higher valuation (or a lower cost) usually exert a higher level of effort. This results can be seen in our Bayesian equilibrium since $(\partial e_1^*)/(\partial V_1) > 0$ for all values of the parameters. The effect of a variation in the opponent's valuation, $(\partial e_1^*)/(\partial V_2)$, will only be positive if $V_1 > V_2$. This means that, while contestant 1 has a higher valuation, increases in V_2

work as an encouragement to fight harder up until the point where valuations are equal, after which it supposes a discouragement. Although the magnitude of the effect depends on α, the sign interestingly only depends on $V_1 > V_2$.

It is also straightforward to prove that $(\partial e_1^*)/\partial p$ is always positive, meaning that the higher the probability of the state of nature where $\psi = \underline{\psi}$, holding everything else constant, the higher the effort of contestant 1 will be. Intuitively, facing a larger p implies an encouragement effect for contestant 1 since it is more likely that the contest will be fair, meaning that the return on investment of effort is higher.

One of the key aspects of our paper is analyzing the response in effort to changes in the fairness parameters. We can see that these follow what one would expect. Increases in either $\overline{\psi}$ or $\underline{\psi}$, ceteris paribus, will always generate increases in e_1^*, since the contestant will believe that the result of the contest is more sensitive to efforts.

Proposition 4. *Any increase in potential fairness for the agent with incomplete information, whether it is through p,* $\overline{\psi}$ *or* $\underline{\psi}$*, increases the effort exerted.*

2.3.2 *Agent with Complete Information*

We now analyze the agent with full information on the realized probability distribution, contestant 2. The comparative statics for this agent are more complex than the one for the agent with incomplete information, since by having access to all the information in the contest, this contestant can make more accurate responses.

It is straightforward to prove (and intuitive) that the effort under the state of *high fairness* is always larger or equal than the one under the *low fairness* scenario. A proof by contradiction can be found in the appendix. Therefore, $\overline{e_2}^* \geq \underline{e}_2^*$ for any possible value of the parameters.

In both cases, an increase in the valuation V_2 induces a high-

$$\frac{\partial \overline{e_2}^*}{\partial V_2} \geq 0 \ if \ \frac{\sqrt{\overline{\psi}}}{\alpha} \geq \frac{(V_1 - V_2)}{(V_1 + V_2)} \quad \text{and} \quad \frac{\partial \underline{e_2}^*}{\partial V_2} \geq 0 \ if \ \frac{\sqrt{\underline{\psi}}}{\alpha} \geq \frac{(V_1 - V_2)}{(V_1 + V_2)}$$

er effort:

The first inequality holds since $\sqrt{(\underline{\psi})}/\alpha \geq 1$, as α is a convex combination of $\sqrt{(\overline{\psi})}$ and $\sqrt{(\underline{\psi})}$ where $\overline{\psi} > \underline{\psi}$, and $(V_1-V_2)/(V_1+V_2) \leq 1$ since all valuations are non-negative. The second inequality holds, because it is a consequence of the inner equilibrium condition (2.1).

When the valuation of the competitor, V_1, increases, the effect in effort is ambiguous:

$$\frac{\partial \overline{e_2}^*}{\partial V_1} \geq 0 \ if \ \frac{\sqrt{\overline{\psi}}}{\alpha} \geq \frac{2V_1}{V_1 + V_2} \quad \text{and} \quad \frac{\partial \underline{e_2}^*}{\partial V_1} \geq 0 \ if \ \frac{\sqrt{\underline{\psi}}}{\alpha} \geq \frac{2V_1}{V_1 + V_2}$$

We know that: $$\frac{\sqrt{\overline{\psi}}}{\alpha} \geq \frac{\sqrt{\underline{\psi}}}{\alpha} \geq \frac{V_1}{V_1 + V_2}$$ from condition (2.1).

If the ratio $(2V_1)/(V_1+V_2)$ is higher than all the terms above, it would cause a decline in the efforts of agent 2 in both scenarios. In the case the ratio is between the first and second term, it would cause an increase in $\overline{e_2}^*$ and a decrease in $\underline{e}_2^*$. And finally, if the ratio is between the second and third term, it would increase the effort of both types of agent 2.

This result can be interestingly compared to the findings in Nti (1999). Consider contestant 1 to be an *underdog*, meaning that they have a significantly lower valuation than contestant 2, so the ratio $V_1/(V_1+V_2)$ is small. Under the setting of rent-seeking with asymmetric valuations studied by Nti (1999), an increase in the underdog's valuation would always lead to a rise in the efforts of both players. However, in our setting of incomplete information, there is a possibility that the same increase of V_1 only increases the effort $\overline{e_2}^*$, and actually decreases $\underline{e}_2^*$. Interestingly, contestant 2's response to a change in p behaves in a

$$\frac{\partial \overline{e_2}^*}{\partial p} \geq 0 \ if \ \frac{\sqrt{\overline{\psi}}}{\alpha} \geq \frac{2V_1}{V_1 + V_2} \quad \text{and} \quad \frac{\partial \underline{e_2}^*}{\partial p} \geq 0 \ if \ \frac{\sqrt{\underline{\psi}}}{\alpha} \geq \frac{2V_1}{V_1 + V_2}$$

similar way, since:
The strategic behavior in the contest strongly depends on the different parameters involved, all contained in α. Taking a deeper look into this recurrent condition, we can extract some valuable insights. First, we know that $\sqrt{\bar{\psi}}/\alpha \geq 1$ and $\sqrt{\underline{\psi}}/\alpha \leq 1$, by construction. At the same time, $(2V_1)/(V_1+V_2)$ will be larger than 1 when $V_1 > V_2$. So, depending on which contestant has the highest valuation, the behavior will be different.

Proposition 5. *The effects on the effort of the agent with complete information of variations in the opponent's valuation or the parameter p are ambiguous and will depend on where the*

$$\frac{\sqrt{\bar{\psi}}}{\alpha} \geq \frac{\sqrt{\underline{\psi}}}{\alpha} \geq \frac{V_1}{V_1+V_2}$$

ratio $(2V_1)/(V_1+V_2)$ lands on the inequality:
One of the most interesting analysis is how the agent with full information responds to a change in the *fairness parameters* $\bar{\psi}$ and $\underline{\psi}$. Fi $\frac{\partial \bar{e}_2^*}{\partial \bar{\psi}}$ and $\frac{\partial \underline{e}_2^*}{\partial \underline{\psi}}$ vill consider what happens when the true fairness p varies, in other words, the signs of the de-

$$\frac{\partial \bar{e}_2^*}{\partial \bar{\psi}} \geq 0 \; if \; \frac{\sqrt{\bar{\psi}}}{\alpha} + \frac{1}{p} \geq \frac{2V_1}{V_1+V_2} \quad \text{and} \quad \frac{\partial \underline{e}_2^*}{\partial \bar{\psi}} \geq 0 \; if \; \frac{\sqrt{\underline{\psi}}}{\alpha} + \frac{1}{1-p} \geq \frac{2V_1}{V_1+V_2}$$

rivatives
And now, we will consider how efforts change when the *unrealized fairness parameter* varies:

$$\frac{\partial \bar{e}_2^*}{\partial \underline{\psi}} \geq 0 \; if \; \frac{\sqrt{\bar{\psi}}}{\alpha} \geq \frac{2V_1}{V_1+V_2} \quad \text{and} \quad \frac{\partial \underline{e}_2^*}{\partial \bar{\psi}} \geq 0 \; if \; \frac{\sqrt{\underline{\psi}}}{\alpha} \geq \frac{2V_1}{V_1+V_2}$$

There is plenty to say about these results. As we can see, the sign of the effect in efforts of a change in the unrealized fairness parameter follows the same condition as what we previously found analyzing the comparative statics regarding V_1 and p. An interesting result is:

$$\frac{\partial \underline{e}_2^*}{\partial \bar{\psi}} \geq 0 \rightarrow \frac{\partial \bar{e}_2^*}{\partial \underline{\psi}} \geq 0 \quad \text{and} \quad \frac{\partial \underline{e}_2^*}{\partial \underline{\psi}} \geq 0 \rightarrow \frac{\partial \underline{e}_2^*}{\partial \bar{\psi}} \geq 0$$

What one would intuitively expect is that any type of *fairness* increase would yield higher efforts from both contestants, since every unit of effort spent increases the winning probabilities marginally more. However, we find that when the ratio $(2V_1)/(V_1+V_2)$ is too high (but still fulfills condition 2.1), an increase in *fairness* can actually *decrease* the efforts of the contestant with full information.

Proposition 6. *An increase in the fairness of a contest, holding everything else constant, always increases the effort of the contestant with incomplete information, but the effect on the effort of the contestant with complete information is ambiguous.*

3 Application to Electoral Competition

We believe the settings of the model proposed in Section II can be used to analyze electoral competition from a novel perspective. Economists, in particular those specialized in the field of Political Economy, have always been interested in how citizens choose their representatives. Some of the initial ideas in this area can be attributed to Hotelling (1929), whose work then led to the development of the Median Voter Theorem and extensions by Downs (1957) and Black (1948), the latter of which is considered by many to be the father of social choice theory.

In electoral competition, candidates try to influence voters in order to increase their probabilities of winning office. The bundle of expenditures spent by each candidate includes a complex list of components, such as mandatory fees, the development of a political plan, salaries during the campaign, and political advertisement, which can be both physical (like political rallies) and through media (radio, TV, internet). All these expenses, which we will call *political pressure*, are the strategy of a candidate in a political competition. Becker (1983) introduces the concept of a *political equilibrium* as the situation where all possible candidates are maximizing their expected payoffs by "*spending their optimal amount on political pressure, given the productivity of their expenditures, and the behavior of the others*."

The popularization of Contest Theory provides a new opportunity for modelling political competition. A simple example can be seen in Corchón (2007), where efforts are measured as the expenditure in advertisement by candidates. Skaperdas and Grofman (1995) remarked the importance of advertising in political competitions and developed a model of "negative campaigning" under the Tullock ratio form. We would like to propose a new model, based on our findings from Section II, to include an extremely important component: heterogeneity in voters. In particular, we believe that the level of spending done by each candidate and their winning probabilities strongly de-

pend on how responsive voters are to *political pressure*.

To illustrate this idea with a real life example, consider the political competition for the presidency of the United States [1], where we can distinguish different types of voters. On the one hand, we have the Red States, which predominantly vote Republican. These States include Nebraska, Kansas, South Dakota, among others. On the other hand, we have the Blue States, which predominantly vote Democrat. These include New York, California, Massachusetts, Vermont, and others. The consistency of electoral results in these states through time (and candidates) tells us that residents have intrinsically different preferences. As a consequence, the return on investment of a single dollar spent by a Democratic Party candidate on a Blue State is significantly higher than on a Red State, and vice versa for a Republican Party candidate.

However, there is also a set of states known as "swing states", which are the most competitive and both parties have similar chances of winning. These include Colorado, Florida, Michigan, among others. These states are the ones that determine who wins the presidential race in most cases. Voters have no intrinsic preference, and a dollar invested here by either candidate can buy practically the same influence.

With this in mind, in our model candidates will be facing two types of voters for simplicity: those who cannot be influenced by any type of political advertisement, which we will call *decided voters*, and those who are persuadable and more responsive to political pressures, which we will call undecided voters. Evidently, within the *decided* category we will have voters who favor each one of the parties. A similar intuition can be found in the work of Baron (1994).

The *fairness parameter* ψ that we introduced in the previous Section will now represent the share of the total voter population that is *undecided*, and therefore sensitive to the political pressure exerted. The remaining $1-\psi$ share of the population will represent those electors who are *decided* and will blindly

1 All electoral data has been found in the "National Archives and Records Administration" from the U.S. Electoral College

vote for a candidate regardless of expenditures. We are also introducing a new parameter γ_i that represents the share of the *decided* voters that favor candidate *i*, so that $\sum_{k=1}^{2}(\gamma_k) = 1$. We assume that this parameter follows a given probability distribution $F(\gamma_i)$ that is not correlated with any other parameter involved in the problem. We are only going to assume that this distribution has an expected value, $E[\gamma_i]$, since its analysis escapes the aim of this paper. The following table summarizes the parameters related to the heterogeneity in voters in our model with two candidates:

Table 1: Parameters of the Electoral Competition Model

Parameter	Description
$\boldsymbol{\psi}$	Share of *undecided* voters
$(\mathbf{1}-\boldsymbol{\psi})\mathbf{E}[\boldsymbol{\gamma}_i]$	Share of *decided* voters favoring i
$(\mathbf{1}-\boldsymbol{\psi})(\mathbf{1}-\mathbf{E}[\boldsymbol{\gamma}_i])$	Share of *decided* voters against i

Therefore, candidate's i expected payoffs function will take the form of:

$$\mathrm{E}[\Pi_i] = \left[(1-\psi)\mathrm{E}[\gamma_i] + \psi\frac{e_i}{\sum_{j=1}^{2} e_j}\right]V_i - e_i \quad \text{for} \quad i \in \{1,2\}$$

Where e_i represents the expenditure in *political pressure* by candidate *i*, and V_i represents the candidate's value of office. For simplicity, we are going to assume that the value of office is the same for the two candidates, so that $V_1 = V_2 = V$. A key characteristic of our model is incompleteness of information. As we previously did, we are going to assume the share of undecided voters can take two possible values $\{\underline{\psi}, \overline{\psi}\}$ with probabilities {1-*p*, *p*} respectively, where $\underline{\psi} < \overline{\psi}$. Candidate 2 knows the true value of ψ, while candidate 1 relies on its probability distribution. This theoretical approach resembles the case of an electoral competition between a known politician and an outsider. One could be a public figure, an incumbent, an ex-president, or someone who has run for office before, so that he has an information advantage, while the other could be a newcomer into politics and is not sure about the share of the population who is *undecided*. As we previously assumed, $E[\gamma_i]$ remains constant in both states of nature for both candidates since its distribution is independent of ψ.

As Becker (1983) formulated, the result of each candidate maximizing their expected payoffs subject to the productivity of their expenditures yields a political equilibrium. In this case, it is the Bayesian equilibrium from our incomplete information setting. The maximization of expected payoffs is exactly similar as in Section II, except for the γ_i parameter, which does not affect the level of optimal expenditure. We will follow the same notation of α ⍰ $(p\sqrt{(\overline{\psi})}+(1-p)\sqrt{(\underline{\psi})})$. Therefore, we find:

$$\frac{\partial \overline{e}_2^*}{\partial \underline{\psi}} \geq 0 \; if \; \frac{\sqrt{\overline{\psi}}}{\alpha} \geq \frac{2V_1}{V_1+V_2} \quad \text{and} \quad \frac{\partial \underline{e}_2^*}{\partial \psi} \geq 0 \; if \; \frac{\sqrt{\underline{\psi}}}{\alpha} \geq \frac{2V_1}{V_1+V_2}$$

With the following winning probabilities:

In the scenario with *high rate of undecided voters*, $\psi = \overline{\psi}$:

$$p_1(e_1^*, \overline{e}_2^*) = (1-\overline{\psi})\mathrm{E}[\gamma_1] + \frac{\alpha\sqrt{\overline{\psi}}}{2}, \; p_2(e_1^*, \overline{e}_2^*) = (1-\overline{\psi})\mathrm{E}[\gamma_2] + \overline{\psi} - \frac{\alpha\sqrt{\overline{\psi}}}{2}$$

In the scenario with *low rate of undecided voters*, $\psi = \underline{\psi}$:

$$p_1\left(e_1^*, \underline{e}_2^*\right) = \left(1-\underline{\psi}\right)\mathrm{E}[\gamma_1] + \frac{\alpha\sqrt{\underline{\psi}}}{2}, \; p_2\left(e_1^*, \underline{e}_2^*\right) = \left(1-\underline{\psi}\right)\mathrm{E}[\gamma_2] + \underline{\psi} - \frac{\alpha\sqrt{\underline{\psi}}}{2}$$

We are interested in analyzing which factors increase a candidate's winning probabilities. **Table 2**, that can be found in Appendix, sums up all the first partial derivatives of p_1 and p_2 for both states of the world with their respective signs. Those with a positive sign imply that any increase in the factor will yield higher winning probabilities for the candidate. The effect of the share of *decided voters* $\mathrm{E}[\gamma_1]$ and $\mathrm{E}[\gamma_2]$ is unambiguous and intuitive. However, the sign of the effect of the share of *undecided voters* on winning probabilities depends on the conditions shown in **Table 2**. Therefore, there is no clear preferences for *fairness* in a contest of this type.

4 Conclusions

Throughout this paper, we have analyzed a contest where participants are uncertain on how sensitive the CSF is with respect to the efforts exerted. In particular, we treated a discrete case of uncertainty through the introduction of an exogenous *fairness parameter* ψ that measures the probability that the winner of the contest is determined by a Tullock ratio instead of by a lottery. First, we solved the case of complete information and found intuitive results that follow traditional contest theory literature. Then, we characterized and analyzed a Bayesian Equilibrium in a two-person contest where a contestant has full information regarding ψ, while the other relies on a probability distribution. Most papers in the field that present a Bayesian Equilibrium mainly consider asymmetries in valuations and costs, so our characterization is novel.

Our results show that comparative statics for the contestant with incomplete information are simple and straightforward, following what has been found in contest theory literature. However, the response of the candidate with complete information to different changes in the parameters depends on the distance between the two beliefs $\{\psi, \overline{\psi}\}$ and the probabilities {1-p, p}. Our key results are that only the contestant with incomplete information always prefers higher levels of *fairness*, while it is ambiguous for the contestant with complete information. In fact, if the player with complete information has significantly lower valuation than the one with incomplete information, any increase in the *fairness* parameter discourages his efforts, regardless of the state of nature.

Finally, we proposed an application to Electoral Competition in which the parameter ψ represents the share of *undecided voters* who can be influenced by political expenditures, while the *decided voters* are divided between the two candidates. A setting of incomplete information, as the one evaluated, resembles the case of an outsider in politics with incomplete information who faces an experienced candidate with complete information regarding the pool of voters. We analyzed the factors that affected the winning probabilities of candidates and found ambiguity regarding the *fairness parameters*.

4.1 *Possible Extensions to the Model*

Although our model is tractable and easy to analyze algebraically, we believe that a more complex approach could be taken in the future, considering a continuous probability distribution for r instead of a discrete one with two values as we did. This could follow what has been done by Fey (2008) with private costs and Gallice (2014) with private valuations. We propose taking the exponent, r, as continuous between a lower bound, a, and an upper bound, b, and providing it with a distribution, F(r), with density f(r). This, which requires more advanced mathematical techniques, would characterize a complete equilibrium. Under this setting, the CSF would be:

$$p_i(e_i, e_{-i}) = \int_a^b f(r)\frac{e_i^r}{\sum_{j=1}^N e_j^r}\,dr$$

We also propose an extension to the model of political competition by including a wider spectrum of types of voters. In addition, a deeper analysis into how the *political pressure* exerted is allocated through voters could complete the model. It is not only important to be aware of the existence of different types of voters among the population (*decided* and *undecided*), but it is also of key importance to be able to identify which specific voter is of which type, in order to allocate the *political pressure* in an effective manner. These kinds of models are currently being developed in forthcoming papers such as Konrad (2017), and we believe it can greatly benefit our understanding of the allocation of *political pressure* and resources in general.

5 References

Amegashie, J. Atsu (2006) A Contest Success Function with a Tractable Noise Parameter. *Public Choice*, 126: 135-144

Anderson, Simon et al. (1998) Rent Seeking with Bounded Rationality: An Analysis of the All-Pay Auction. Journal of Political Economy, 106: 828 – 853

Bahard, Eyal & Nitzan, Shmuel (2008) Contest Efforts in Light of Behavioural Considerations. The Economic Journal, 118: 2047 – 2059

Baron, David (1994) Electoral Competition with Informed and Uninformed Voters. The American Political Science Review, 88: 33 – 47

Becker, Gary (1983) A Theory of Competition Among Pressure Groups for Political Influence. The Quarterly Journal of Economics, 98: 371 – 400

Black, Duncan (1948) On the Rationale of Group Decision-Making. Journal of Political Economy, 56: 23 – 34

Corchón, Luis (2007) The Theory of Contests: a Survey. Review of Economic Design 11: 69 – 100

Corchón, Luis & Dahm, Matthias (2010) Foundations for Con test Success Functions. Economic Theory, 43: 81 – 98

Downs, Anthony (1957) An Economic Theory of Political Action in a Democracy. The Journal of Political Economy, 65: 135 – 150

Einy, Ezra et al. (2015) Equilibrium Existence in Tullock Contests with Incomplete Information. Journal of Mathematical Economics, 61: 241 – 245

Fey, Mark (2008) Rent-seeking Contests with Incomplete Information. Public Choice, 135: 225-236

Franke, Jörg, et al.(2014) Revenue Maximizing Head Starts in Contests. Ruhr Economic Papers #524

Gallice, Andrea (2014) Rent-seeking Contests With Private Valuations. Carlo Alberto Notebooks Working Papers, 390

Hirshleifer, Jack (1989) Conflict and Rent-seeking Success Functions: Ratio vs. Difference Models of Relative Success. Public Choice, 63: 101-112

Hotelling, Harold (1929). Stability in Competition. The Economic Journal, 39: 41 – 57

Hurley, Terrance & Shogren, Jason (1998) Asymmetric Information Contests. European Journal of Political Economy, 14, 645 – 665

Jia, Hao (2008) A Stochastic Derivation of the Ratio Form of Contest Success Functions. Public Choice, 135: 125 – 130

Jia, Hao, et al. (2013) Contest functions: Theoretical Foundations and Issues in Estimation. International Journal of Industrial Organization, 31: 211 – 222

Konrad, Kai, et al. (2017) Targeted Campaign Competition, Loyal Voters, and Supermajorities. Journal of Mathematical Economics, 71, 49 – 62

Malueg, David & Yates, Andrew (2004) Rent Seeking with Private Values. Public Choice, 119: 161 – 178

Nitzan, Shmuel (1991) Collective Rent Dissipation. The Economic Journal, 101: 1522 – 1534

Nti, Kofi (1999) Rent-seeking with Asymmetric Valuations. Public Choice, 98: 415

Ryvkin, Dmitry (2010) Contests with Private Costs: Beyond Two Players. European Journal of Political Economy, 26, 558 – 567

Singh, Nirvikar & Wittman, Donald (1988) Economic Contests with Incomplete Information and Optimal Contest Design. Management Science, 34, 528 – 540

Skaperdas, Stergios (1996) Contest Success Functions. Economic Theory, 7: 283 – 290

Skaperdas, Stergios & Grofman, Bernard (1995) Modeling Negative Campaigning. The American Political Science Review, 89: 49 – 61

Tullock, Gordon (1967) The Welfare Costs of Tariffs, Monopolies, and Theft. Western Economic Journal, 5:3, p. 224

Tullock, Gordon, (1980) Efficient Rent-seeking. In Buchanan JM, Tollison RD, Tullock G (eds) Towards a theory of a rent-seeking society. 97-112. Texas A&M University Press, College Station.

6 Appendix and Tables

2.2 Bayesian Equilibrium

Solution of the equilibrium – Algebra

The three first order conditions are:

$$\text{(i)}\ \underline{e_2} = \sqrt{\underline{\psi} V_2 e_1} - e_1 \qquad \text{(ii)}\ \overline{e_2} = \sqrt{\overline{\psi} V_2 e_1} - e_1 \qquad \text{(iii)}\ p\overline{\psi}\frac{\overline{e_2}}{(e_1+\overline{e_2})^2}V_1 + (1-p)\underline{\psi}\frac{\underline{e_2}}{(e_1+\underline{e_2})^2}V_1 = 1$$

Substituting (i) and (ii) and (iii):

$$p\overline{\psi}\frac{\sqrt{\overline{\psi}V_2e_1}-e_1}{\left(e_1+\sqrt{\overline{\psi}V_2e_1}-e_1\right)^2}V_1 + (1-p)\underline{\psi}\frac{\sqrt{\underline{\psi}V_2e_1}-e_1}{\left(e_1+\sqrt{\underline{\psi}V_2e_1}-e_1\right)^2}V_1 = 1$$

$$p\frac{\sqrt{\overline{\psi}V_2e_1}-e_1}{e_1V_2}V_1 + (1-p)\frac{\sqrt{\underline{\psi}V_2e_1}-e_1}{e_1V_2}V_1 = 1$$

$$\frac{pV_1\sqrt{\overline{\psi}}}{\sqrt{e_1V_2}} + \frac{(1-p)V_1\sqrt{\underline{\psi}}}{\sqrt{e_1V_2}} - \frac{pV_1}{V_2} - \frac{(1-p)V_1}{V_2} = 1$$

$$\frac{\left(p\sqrt{\overline{\psi}}+(1-p)\sqrt{\underline{\psi}}\right)V_1}{\sqrt{e_1V_2}} - \frac{V_1}{V_2} = 1$$

$$\left(p\sqrt{\overline{\psi}} + (1-p)\sqrt{\underline{\psi}}\right)V_1\sqrt{V_2} = \sqrt{e_1}(V_1+V_2)$$

Therefore:

$$e_1^* = \frac{\left(p\sqrt{\overline{\psi}} + (1-p)\sqrt{\underline{\psi}}\right)^2 V_1^2V_2}{(V_1+V_2)^2}$$

Substituting e_1^* into (i) and (ii) gives us:

$$\overline{e_2}^* = \frac{\left(p\sqrt{\overline{\psi}}+(1-p)\sqrt{\underline{\psi}}\right)V_1V_2}{(V_1+V_2)}\left[\sqrt{\overline{\psi}} - \frac{\left(p\sqrt{\overline{\psi}}+(1-p)\sqrt{\underline{\psi}}\right)V_1}{(V_1+V_2)}\right] \qquad \underline{e_2}^* = \frac{\left(p\sqrt{\overline{\psi}}+(1-p)\sqrt{\underline{\psi}}\right)V_1V_2}{(V_1+V_2)}\left[\sqrt{\underline{\psi}} - \frac{\left(p\sqrt{\overline{\psi}}+(1-p)\sqrt{\underline{\psi}}\right)V_1}{(V_1+V_2)}\right]$$

Proof of the sufficient condition for the existence of the Bayesian Equilibrium

The two conditions needed for the existence of the Bayesian Equilibrium are:

$$\overline{e_2}^* \geq 0 \leftrightarrow \frac{\sqrt{\overline{\psi}}}{\alpha} \geq \frac{V_1}{(V_1+V_2)} \text{ and } \underline{e_2}^* \geq 0 \leftrightarrow \frac{\sqrt{\underline{\psi}}}{\alpha} \geq \frac{V_1}{(V_1+V_2)}$$

The latter is sufficient for both to hold, since:

$$\overline{\psi} > \underline{\psi} \text{ by assumption}$$

$$\sqrt{\overline{\psi}} > \sqrt{\underline{\psi}} \leftrightarrow \frac{\sqrt{\overline{\psi}}}{\alpha} > \frac{\sqrt{\underline{\psi}}}{\alpha}$$

Since α is by definition non-negative. Therefore, if $\sqrt{(\underline{\psi})}/\alpha \geq V_1/(V_1+V_2)$ holds, then $\sqrt{(\overline{\psi})}/\alpha \geq V_1/(V_1+V_2)$ will hold as well.

2.3 Comparative Statics and Analysis

2.3.1 Agent with Incomplete Information

$$\frac{\partial e_1^*}{\partial V_1} = \frac{2\alpha^2V_1V_2(V_1+V_2)^2 - 2(V_1+V_2)\alpha^2V_1^2V_2}{(V_1+V_2)^4} = \frac{2\alpha^2V_1V_2^2}{(V_1+V_2)^3} \geq 0$$

$$\frac{\partial e_1^*}{\partial V_2} = \frac{\alpha^2V_1^2(V_1+V_2)^2 - 2(V_1+V_2)\alpha^2V_1^2V_2}{(V_1+V_2)^4} = \frac{\alpha^2V_1^2(V_1-V_2)}{(V_1+V_2)^3} \geq 0 \ if\ V_1 \geq V_2$$

$$\frac{\partial e_1^*}{\partial p} = \frac{2\alpha V_1^2V_2}{(V_1+V_2)^2}\left(\sqrt{\overline{\psi}} - \sqrt{\underline{\psi}}\right) \geq 0$$

$$\frac{\partial e_1^*}{\partial \overline{\psi}} = \frac{2\alpha V_1^2V_2p\frac{1}{2}\left(\overline{\psi}\right)^{-\frac{1}{2}}}{(V_1+V_2)^2} = \frac{\alpha V_1^2V_2p}{(V_1+V_2)^2\sqrt{\overline{\psi}}} \geq 0$$

$$\frac{\partial e_1^*}{\partial \underline{\psi}} = \frac{2\alpha V_1^2V_2(1-p)\frac{1}{2}\left(\underline{\psi}\right)^{-\frac{1}{2}}}{(V_1+V_2)^2} = \frac{\alpha V_1^2V_2(1-p)}{(V_1+V_2)^2\sqrt{\underline{\psi}}} \geq 0$$

2.3.2 Agent with Complete Information

Effort is always larger with the high fairness parameter

We are going to perform a proof by contradiction. Assume $\underline{\psi} < \overline{\psi}$, each one respectively associated with an effort $\underline{e}_2^*$, $\overline{e_2}^*$. If we consider that $\underline{e}_2^* > \overline{e_2}^*$, then:
Which is a contradiction, since we first assumed that $\underline{\psi} < \overline{\psi}$.

$$\frac{\alpha V_1 V_2}{(V_1+V_2)}\left[\sqrt{\underline{\psi}} - \frac{\alpha V_1}{(V_1+V_2)}\right] > \frac{\alpha V_1 V_2}{(V_1+V_2)}\left[\sqrt{\overline{\psi}} - \frac{\alpha V_1}{(V_1+V_2)}\right]$$

$$\frac{\alpha V_1 V_2 \sqrt{\underline{\psi}}}{(V_1+V_2)} > \frac{\alpha V_1 V_2 \sqrt{\overline{\psi}}}{(V_1+V_2)}$$

$$\sqrt{\underline{\psi}} > \sqrt{\overline{\psi}}$$

Hence, it is proved that a higher level of *fairness* always leads to a higher effort.

Comparative Statics for $\overline{e_2}^*$

$\frac{\partial \overline{e_2}^*}{\partial V_2} = \frac{\sqrt{\overline{\psi}}\alpha V_1^2}{(V_1+V_2)^2} - \frac{\alpha^2 V_1^2 (V_1+V_2)(V_1-V_2)}{(V_1+V_2)^4}$ will be positive if $\frac{\sqrt{\overline{\psi}}}{\alpha} > \frac{(V_1-V_2)}{(V_1+V_2)}$. Since $V_i \geq 0\ \forall i$, $\frac{(V_1-V_2)}{(V_1+V_2)} \leq 1$, as α is a convex combination of $\sqrt{\overline{\psi}}$ and $\sqrt{\underline{\psi}}$, the term $\frac{\sqrt{\overline{\psi}}}{\alpha} \geq 1$. Hence, $\frac{\partial \overline{e_2}^*}{\partial V_2} \geq 0$

$$\frac{\partial \overline{e_2}^*}{\partial V_1} = \frac{\sqrt{\overline{\psi}}\alpha V_2^2}{(V_1+V_2)^2} - \frac{2\alpha^2 V_1 V_2^2 (V_1+V_2)}{(V_1+V_2)^4} > 0 \ if\ \frac{\sqrt{\overline{\psi}}}{\alpha} > \frac{2V_1}{V_1+V_2}$$

$$\frac{\partial \overline{e_2}^*}{\partial p} = \frac{\sqrt{\overline{\psi}} V_1 V_2}{(V_1+V_2)}\left(\sqrt{\overline{\psi}} - \sqrt{\underline{\psi}}\right) - \frac{2\alpha V_1^2 V_2}{(V_1+V_2)^2}\left(\sqrt{\overline{\psi}} - \sqrt{\underline{\psi}}\right) > 0 \ if\ \frac{\sqrt{\overline{\psi}}}{\alpha} > \frac{2V_1}{V_1+V_2}$$

$$\frac{\partial \overline{e_2}^*}{\partial \overline{\psi}} = \frac{p V_1 V_2}{(V_1+V_2)} + \frac{(1-p)\sqrt{\underline{\psi}} V_1 V_2}{2(V_1+V_2)\sqrt{\overline{\psi}}} - \frac{\alpha V_1^2 V_2 p}{(V_1+V_2)^2\sqrt{\overline{\psi}}} = \frac{\left(2p\sqrt{\overline{\psi}} + (1-p)\sqrt{\underline{\psi}}\right) V_1 V_2}{2(V_1+V_2)\sqrt{\overline{\psi}}} - \frac{\alpha V_1^2 V_2 p}{(V_1+V_2)^2\sqrt{\overline{\psi}}}$$

$$= \frac{\left(p\sqrt{\overline{\psi}} + \alpha\right) V_1 V_2}{2(V_1+V_2)\sqrt{\overline{\psi}}} - \frac{\alpha V_1^2 V_2 p}{(V_1+V_2)^2\sqrt{\overline{\psi}}} > 0 \ if\ \frac{\sqrt{\overline{\psi}}}{\alpha} + \frac{1}{p} > \frac{2V_1}{V_1+V_2}$$

$$\frac{\partial \overline{e_2}^*}{\partial \underline{\psi}} = \frac{(1-p)\sqrt{\overline{\psi}} V_1 V_2}{2(V_1+V_2)\sqrt{\underline{\psi}}} - \frac{\alpha V_1^2 V_2 (1-p)}{(V_1+V_2)^2\sqrt{\underline{\psi}}} > 0 \ if\ \frac{\sqrt{\overline{\psi}}}{\alpha} > \frac{2V_1}{V_1+V_2}$$

Comparative Statics for $\underline{e}_2^*$

$\frac{\partial \underline{e_2}^*}{\partial V_2} = \frac{\sqrt{\underline{\psi}}\alpha V_1^2}{(V_1+V_2)^2} - \frac{\alpha^2 V_1^2 (V_1+V_2)(V_1-V_2)}{(V_1+V_2)^4}$ will be positive if $\frac{\sqrt{\underline{\psi}}}{\alpha} > \frac{(V_1-V_2)}{(V_1+V_2)}$. This condition always holds because of the equilibrium condition (2.1).

$$\frac{\partial \underline{e_2}^*}{\partial V_1} = \frac{\sqrt{\underline{\psi}}\alpha V_2^2}{(V_1+V_2)^2} - \frac{2\alpha^2 V_1 V_2^2 (V_1+V_2)}{(V_1+V_2)^4} > 0 \ if\ \frac{\sqrt{\underline{\psi}}}{\alpha} > \frac{2V_1}{V_1+V_2}$$

$$\frac{\partial \underline{e_2}^*}{\partial p} = \frac{\sqrt{\underline{\psi}} V_1 V_2}{(V_1+V_2)}\left(\sqrt{\overline{\psi}} - \sqrt{\underline{\psi}}\right) - \frac{2\alpha V_1^2 V_2}{(V_1+V_2)^2}\left(\sqrt{\overline{\psi}} - \sqrt{\underline{\psi}}\right) > 0 \ if\ \frac{\sqrt{\underline{\psi}}}{\alpha} > \frac{2V_1}{V_1+V_2}$$

$$\frac{\partial \underline{e_2}^*}{\partial \overline{\psi}} = \frac{p\sqrt{\underline{\psi}} V_1 V_2}{2(V_1+V_2)\sqrt{\overline{\psi}}} - \frac{\alpha V_1^2 V_2 p}{(V_1+V_2)^2\sqrt{\overline{\psi}}} > 0 \ if\ \frac{\sqrt{\underline{\psi}}}{\alpha} > \frac{2V_1}{V_1+V_2}$$

$$\frac{\partial \underline{e_2}^*}{\partial \underline{\psi}} = \frac{p\sqrt{\overline{\psi}} V_1 V_2}{(V_1+V_2)\sqrt{\underline{\psi}}} + \frac{(1-p) V_1 V_2}{(V_1+V_2)} - \frac{\alpha V_1^2 V_2 (1-p)}{(V_1+V_2)^2\sqrt{\underline{\psi}}} = \frac{\left(p\sqrt{\overline{\psi}} + 2(1-p)\sqrt{\underline{\psi}}\right) V_1 V_2}{2(V_1+V_2)\sqrt{\underline{\psi}}} - \frac{\alpha V_1^2 V_2 (1-p)}{(V_1+V_2)^2\sqrt{\underline{\psi}}}$$

$$= \frac{\left(\alpha + (1-p)\sqrt{\underline{\psi}}\right) V_1 V_2}{2(V_1+V_2)\sqrt{\underline{\psi}}} - \frac{\alpha V_1^2 V_2 (1-p)}{(V_1+V_2)^2\sqrt{\underline{\psi}}} > 0 \ if\ \frac{\sqrt{\underline{\psi}}}{\alpha} + \frac{1}{1-p} > \frac{2V_1}{V_1+V_2}$$

3 Applications to Electoral Competition

Table 2. First partial derivatives of p_1 and p_2 for both states of the world

	If $\psi = \overline{\psi}$		If $\psi = \underline{\psi}$	
	$p_1(e_1^*, \overline{e}_2^*)$	$p_2(e_1^*, \overline{e}_2^*)$	$p_1(e_1^*, \underline{e_2^*})$	$p_2(e_1^*, \underline{e_2^*})$
$\frac{\partial}{\partial p}$	≥ 0	≤ 0	≤ 0	≥ 0
$\frac{\partial}{\partial \overline{\psi}}$	≥ 0 if $\frac{p\sqrt{\overline{\psi}+\alpha}}{4\sqrt{\overline{\psi}}} \geq \mathrm{E}[\gamma_1]$	≥ 0 if $\frac{p\sqrt{\overline{\psi}+\alpha}}{4\sqrt{\overline{\psi}}} \leq \mathrm{E}[\gamma_1]$	≥ 0	≤ 0
$\frac{\partial}{\partial \underline{\psi}}$	≥ 0	≤ 0	≥ 0 if $\frac{(1-p)\sqrt{\underline{\psi}+\alpha}}{4\sqrt{\underline{\psi}}} \geq \mathrm{E}[\gamma_1]$	≥ 0 if $\frac{(1-p)\sqrt{\underline{\psi}+\alpha}}{4\sqrt{\underline{\psi}}} \leq \mathrm{E}[\gamma_1]$
$\frac{\partial}{\partial \mathrm{E}[\gamma_1]}$	≥ 0	≤ 0	≥ 0	≤ 0
$\frac{\partial}{\partial \mathrm{E}[\gamma_2]}$	≤ 0	≥ 0	≤ 0	≥ 0

[THIS PAGE INTENTIONALLY LEFT BLANK]